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Multi-item Inventory model with Two Price-breaks under Multiple Recovery and Procurement Set-ups

***Jain, M., * Sharma, G. C. and **Singh, Ruchita Raj**

*Institute of Basic Science, Khandari, Agra 282002, India.

**Department of Mathematics, St. John's College, Agra 282002, India.

*E-Mail: madhujain@bsnl.in, gokulchandra@bsnl.in

**ruchitarajsingh_13@yahoo.com

Abstract

In this paper an effort has been made to analyze an inventory model with multiple items for multiple recovery and procurement setups with two price breaks in the purchase price. These recovery and procurement setups are engaged in collecting used items from customers as well as purchasing of new items. The used items are worked upon again and made equivalent to the new ones. Further the model deals with two price breaks at which discounts are offered for large purchase. The total cost for the model is incurred and an algorithm to find the optimum order quantity is presented. Thus, the optimum order quantity that minimizes the total cost is obtained. Finally the model is illustrated numerically in order to validate the analytical results.

Keywords: Inventory, Multi-item, Two price-breaks, Multiple set-up, Optimum order quantity, Recoverable item, Fuzzy ordering cost.

AMS subject classification: 90B05

1. Introduction

One of the major advantages of inventory is the economics of purchase with bulk purchasing. The sellers offer discounts on mass purchasing and the discount increases with the increase in the quantity purchased. These discounts cut the cost of material to a great extent. Apart from reducing the purchase cost, bulk purchasing also helps in dropping the cost of transportation and adds to fast and adequate service to the customers. It even works better when there are multiple items for purchase as large discounts can be thought on each type of item simultaneously. In the present investigation, we analyze inventory model with multiple items, two price breaks, and multiple recovery and procurement setups. There exist two units with multiple setups comprising of recovery unit and purchase unit responsible for collecting the used items and purchasing the new items respectively. The used items are converted into "processed recovered items" by working on them and making them reusable and free from all defects. The total demands of all sorts of items are fulfilled partially by new items and partially by processed recovered items. Such a procedure not only helps in reducing the cost but also helps in recycling the product.

The purchase price of new item is not constant but discount is offered on it and is proportional to the increase in quantity purchased. In the present paper, we assume that the price of the item changes at two points which are called break points. These points are independent of the type of product but depend only on the quantity purchased. Here inventory is basically composed of three types of product which are the unprocessed recovered items, processed recovered items and the new items. The key concerned objective is to find the optimum order quantity which minimizes the total cost for the entire model. The concepts of recovery and multiple items with two price-breaks make our model more versatile and realistic in comparison to earlier existing models in the literature.

Business strategies are being modified every hour to cope up with the cut throat competition of today's world. Earlier when the availability of raw material was enormous, the production as well as purchase of new items was easier. Now when the resources are declining, it is economical to recycle the product. Thus in the present business era, the focus is on recovering the used commodities. Integrated inventory models were developed by Goyal and Gupta (1989) representing the vendor-buyer co-ordination. Minner (2003) studied multiple-supplier inventory model focusing on issues like operational flexibility in global sourcing. Lot sizing inventory system with product recovery was proposed by Teunter (2004). The use of third party logistic provider was proposed by Tang and Lee (2005). The complex structure of joint manufacturing and remanufacturing system was addressed by Bhattacharya et al. (2006), Corbacioglu and Laon (2007), Tang et al. (2007) and Chung et al. (2008). An optimal policy for multiple recovery and procurement setups was presented by Sarker et al. (2009). They discussed finite and infinite cases for procurement/production rate. An inventory control model with consideration of remanufacturing and product life cycle was considered by Hsueh (2011).

The appropriate discount at huge purchasing attracts both the buyer and the retailers to utilize their limited resources optimally. Discount at bulk purchase benefits both the seller as well as the purchaser. The sellers get large sum of money altogether which enhances their capital while the purchaser gets the commodity at a lower price which in turn reduces his per capita cost. There may be single or multiple price breaks depending upon the negotiation between both the parties. The price falls at all the break points as such there exists a wider horizon for the purchaser to decide the quantity to be purchased in order to make optimum utilization of his money. Britney et al. (1983) analyzed the full fixed cost recovery lot sizing based on supplier's price-quantity discount schedule. A buyer-dependent quantity discount perspective was presented by Martin (1993). Many other constraints of inventory management may be addressed along with price-discount like limited resource, lead-time, shortage and just in time (JIT) schedule. The concept of quantity discount along with JIT, partial backlogging, variable lead time were proposed by Schniederjans and Cao (2000), Papachristos and Skouri (2003) and Chang et al. (2006).

There can be more than one type of discount offered to the purchaser. Similarly vendors and producers may offer discount simultaneously which results in non-linear supply chain management (SCM). Tsai (2007) and Shin and Benton (2007) studied the SCM models including non-linear SCM and SCM co-ordination with discount respectively. The inventory management is more challenging while considering multiple items with quantity discount. There are various demands for different merchandise and one has to make a balance between the selling price and supplier's offer. Multi-item inventory system with quantity discounts was examined by Haksever and Moussourakis (2008). Inventory models including permissible delay in payments and centralized purchasing decision were provided by Sana and Chaudhuri (2008) and Munson and Hu (2010), respectively. The concept of powerful buyer who has increased power over the supplier was studied by Lin (2010). Both imperfect quality and quantity discount were considered to obtain the optimum ordering policy. An inventory model with random discount offerings was considered by Tajbakhsh et al. (2011). Discount pricing for a family of items was evaluated by Ke and Bookbinder (2012).

The logistic refers to the purchasing of the raw material, followed by production and the distribution of the material. The accurate number of deliveries, optimal cost of product, optimal ordering cost, setup cost and optimal lot size cannot be found precisely. These random parameters can be tackled using fuzzy logic. Zadah (1965) introduced fuzzy set in a universal scenario. The fuzzy set can help management to exploit the full potential of logistic process. Chen (1985) studied some operations on fuzzy number useful to develop a model. EOQ models with fuzzy sets were developed by Kao and Hsu (2002), Katagiri and Ishri (2002), Chen (2003) and Chang (2004) and many more. The recent works done on fuzzy set in inventory system include Halim et al. (2008), Shiang (2008) and Vijayan (2009).

This investigation presents an inventory model with multiple-items, two price breaks with multiple recovery and procurement setups. The optimum order quantity is derived in an environment where the price of the new items fall at two points. The contents of the paper have been organized in the following manner. The assumptions and notations are presented in section 2. The inventory model with multiple-items, two price breaks with multiple recovery and procurement setups is developed in section 3. The crisp and fuzzy mathematical models are described in sections 4 and 5 respectively. The algorithm to find the optimum order quantity is presented in section 6. The analytical results of the model have been numerically illustrated in section 7. Finally the conclusions are drawn in section 8.

2. Assumptions and Notations

We consider multi-item inventory model with multiple setup for recovery and procurements. The total demand if fulfilled by reworking on the used items and making them as good as new ones as well as by procuring new items. The inventory model considers multiple items. The recovery set-up deals with the reworking of used items while the procurement set-up is involved in purchasing new items. The demand and repair capacity are assumed to be greater than the recovery rate. Further there are no shortages in inventory in the complete planning horizon. It is assumed that discount is offered for different ranges of the purchased quantity. The point at which the price of item falls is called the point of price-break. There is also provision for two price-breaks in the model. Here we assume that the purchase price of the new item is not constant; but it decreases with the increase in the quantity purchased. The discount offered is uniform irrespective of the type of item but depends only on the quantity purchased. The price for i^{th} item P_{ij} (i=1, 2,….n), (j=1,2,3) is offered based on quantity purchased as follows:

Price of i^{th} item (i=1,2,….n)	**Range of the purchased quantity**
P_{i1}	$b_0 \le Q_{i1} < b_1$
P_{i2}	$b_1 \le Q_{i2} < b_2$
P_{i3}	$Q_{i3} \ge b_2$

Here $P_{i1} > P_{i2} > P_{i3}$.

The following notations are used to describe the model.

m_i Number of recoveries for i^{th} item.

n_i Number of procurements for i^{th} item.

D_i Demand of i^{th} item with $D_i > r_i$.

r_i Recovery rate of i^{th} item.

λ_i Repair capacity of i^{th} item, which is known and finite with $\lambda_i > r_i$.

A_i Ordering cost for i^{th} new item (Rs./ order) .

A_{si} Set-up cost for recovery process of i^{th} new item (Rs./order).

β_i Processed inventory consumption rate for i^{th} item.

h_{ri} Holding cost of recovered i^{th} item.

h_{si} Holding cost of i^{th} serviceable item.

Q_i Order quantity for the newly procured i^{th} item.

R_i Inventory level of recoverable i^{th} item to start the recovery process.

T Cycle time of the entire model.

t_l Time to collect the items from the customers.

t_r Batch cycle time for a set-up.

T The cycle time for one order.

P_{ij} The price for i^{th} item (j=1, 2, 3).

I The cost of carrying one rupee into the inventory for one year.

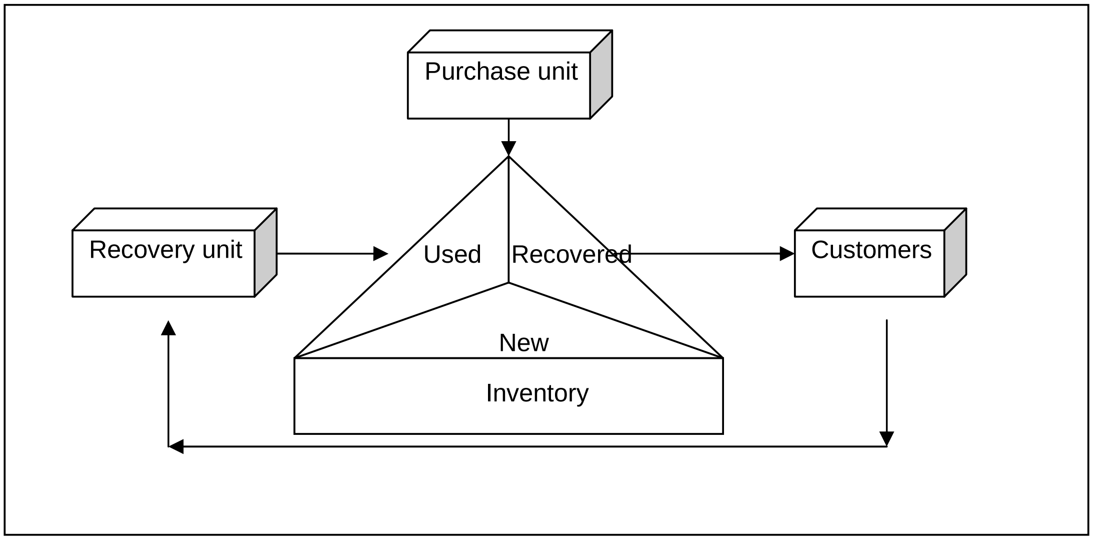
b_1, b_2 The points of price break.

$\tilde{A}_{si}$ The fuzzy set-up cost for recovery process of i^{th} new item (Rs./order).

$\tilde{A}_i$ The fuzzy ordering cost for i^{th} new item (Rs./ order) .

3. Model Description

The present inventory model deals with a system in which there are two subsystems. The first subsystem is the recovery unit while other subsystem is the purchase unit. The function of the recovery unit is to collect the used items from the customers then the restoration work is done on them to make them as good as new ones. These are referred as recovered items. The purchase unit is responsible for the purchase of new items. Thus, the items from both the units together form the inventory system comprising of three types of items i.e. used items, recovered items and the new items (see fig. 1).



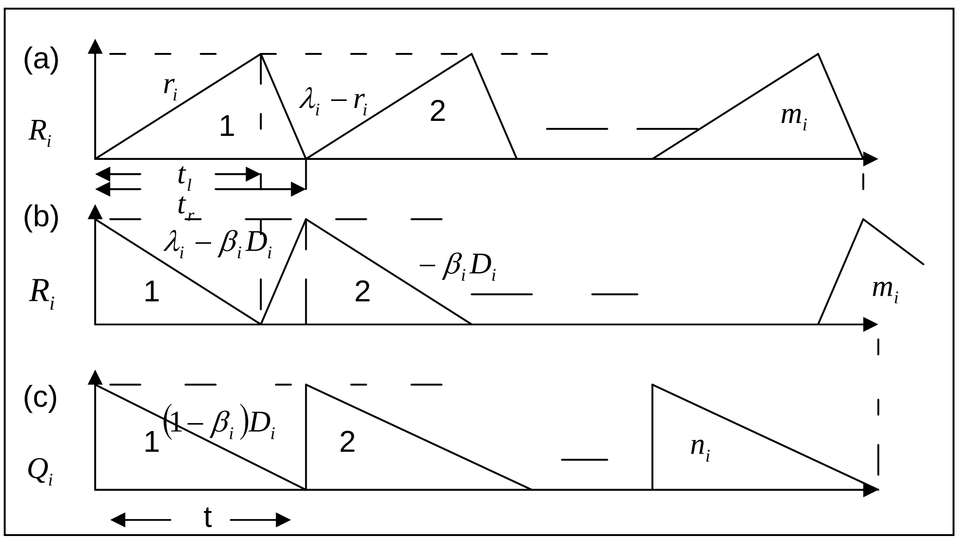


Fig. 2 The inventory levels for unprocessed, processed and new items

The work of the purchase unit is far challenging in the present scenario. In order to encourage large purchase order, the sellers offer some discounts. Thus, the purchase price of the different types of items is not constant but decreases with the increase in the quantity purchased. The present model has two price-breaks. The price of the items fall at the points b_1 and b_2. These are called points of price breaks. The discount is offered at two values of the quantity ordered. If $b_0 \leq Q_{i1} < b_1$ then the price of the new items is P_{i1} but if the quantity to be ordered increases and $b_1 \leq Q_{i2} < b_2$ then a discount is offered and the price falls to P_{i2}. If the quantity ordered increases beyond it with $Q_{i3} \geq b_2$ then larger discount is offered and the price of the new product is P_{i3}.

The inventory levels for unprocessed recovered items, processed recovered and new items are illustrated in fig. 2 (a), (b) and (c) respectively. There are multiple recovery set-ups and multiple purchase orders of material. For this purpose the set-up cost is considered low. The inventory for the unprocessed recovered items increases at a rate r_i, reaches a maximum level R_i and then depletes at a rate $(\lambda_i - r_i)$; thus one recovery cycle is completed. Then the next cycle starts after this and so on. The processed recovered inventory increases at a rate $(\lambda_i - \beta_i D_i)$ and decreases at a rate $-\beta_i D_i$. The purchased quantity Q_i of new items adds up to the inventory. It reduces at a rate $(1-\beta_i)D_i$ till it reaches zero and then after the next procurement is done.

4. The Economic Order Quantity Model

Since R_i is the inventory level for ith (i=1, 2, ----n) recoverable item to start a recovery process with λ_i as the capacity, therefore the time required i.e. the batch cycle time is given as

$$t_r = \frac{R_i \lambda_i}{r_i(\lambda_i - r_i)} \tag{1}$$

where r_i is the rate at which the inventory for ith recoverable item builds up and the inventory depletes at a rate $(\lambda_i - r_i)$. The total cycle time for m_i batches is

$$T = m_i t_r \tag{2}$$

Also the total cycle time for n_i orders is

$$T = n_i t \tag{3}$$

where $t = \frac{Q_i}{(1-\beta_i)D_i}$ (4)

is the cycle time for one order. On equating the two values of T given in eqs (3) and (4), we obtain

$$Q_i = \frac{m_i R_i \lambda_i (D_i - r_i)}{r_i n_i (\lambda_i - r_i)} \tag{5}$$

There are three types of inventory in the system. These are

(i) The unprocessed recovered inventory which is collected from the customers.
(ii) The processed recovered inventory which are as good as new ones.
(iii) The newly procured inventory.

Apart from the holding cost, the set-up costs are incurred for m_i recovery set-ups and for n_i procurements of new items. Finally the purchase cost is included in the total cost for the model. The different costs incurred are as follows.

(a) Purchase cost of ith item (per unit time)

$$C_P = D_i P_{ij} \quad ,(j = 1,2,3) \tag{6}$$

(b) Annual holding cost associated with set-ups (per unit time)

$$C_H = \frac{1}{2} A_i I \tag{7}$$

(c) The carrying cost of recovered unprocessed inventory

$$C_{RU} = \frac{R_i h_{ri} T}{2} \tag{8}$$

(d) The set-up cost for m_i recovery set-ups

$$C_S = A_{si} m_i \tag{9}$$

(e) The inventory carrying cost of recovered processed items

$$C_{RP} = \frac{R_i h_{si} T}{2} \tag{10}$$

(f) The set-up cost for the new items

$$C_{SN} = n_i A_i \tag{11}$$

The inventory carrying cost of the newly procured items

$$C_{NP} = \frac{n_i t Q_i IP_{ij}}{2} = \frac{T Q_i IP_{ij}}{2} \tag{12}$$

The total cost for the ith (i=1, 2,….n) item, per unit time is given as

$$E\{TC_i\} = D_i P_{ij} + \frac{1}{2} A_i I + \frac{R_i h_{ri}}{2} + \frac{A_{si} m_i}{T} + \frac{R_i h_{si}}{2} + \frac{A_i n_i}{T} + \frac{Q_i IP_{ij}}{2} \tag{13}$$

On substituting the value for T using eqs (3) and (4) in (13), we get the total cost as a function of Q_i for ith item as

$$E\{TC(Q_i)\} = D_i P_{ij} + \frac{1}{2} A_i I + \frac{R_i h_{ri}}{2} + \frac{A_{si} m_i (1-\beta_i) D_i}{n_i Q_i} + \frac{R_i h_{si}}{2} + \frac{A_i (1-\beta_i) D_i}{Q_i} + \frac{Q_i IP_{ij}}{2} \tag{14}$$

The optimum order quantity can be obtained by differentiating partially equation (14) w. r. t. Q_i and equating to zero. Thus, the optimum order quantity for ith item is given by

$$Q_{ij}^* = Q_i^* = \sqrt{\frac{2D_i(1-\beta_i)(A_{si} m_i + A_i n_i)}{n_i IP_{ij}}} \qquad (j = 1,2,3) \tag{15}$$

The optimum total cost comprising of all types of items is given by

$$E\{TC(Q_{ij}^*)\} = \sum_{i=1}^{n} D_i P_{ij} + A_i \left(\frac{I}{2} + \frac{(1-\beta_i) D_i}{Q_{ij}^*} \right) + R_i \left(\frac{h_{ri} + h_{si}}{2} \right) + \frac{A_{si} m_i (1-\beta_i) D_i}{n_i Q_{ij}^*} + \frac{Q_{ij}^* IP_{ij}}{2} \tag{16}$$

$E\{TC(Q_{ij}^*)\}$ is the optimum total cost when the purchased quantity Q_{ij}^*, $(j = 1,2,3)$ lies in the range $b_{j-1} \le Q_{ij} \le b_j$. $E\{TC(Q_{ij}^*)\}$ is the optimum cost as we see that $\frac{\partial^2 E\{TC(Q_{ij}^*)\}}{\partial Q_{ij}^2} = IP_{ij} \sqrt{\frac{n_i IP_{ij}}{2(1-\beta_i) D_i (A_{si} m_i + A_i n_i)}} > 0$.

By using the values Q_{i1}^*, Q_{i2}^* and Q_{i3}^*, in equation (16) we may obtain the optimum costs as $E\{TC(Q_{i1}^*)\}$, $E\{TC(Q_{i2}^*)\}$ and $E\{TC(Q_{i3}^*)\}$.

5. Fuzzy Inventory Model

We have also analyzed the inventory model in a fuzzy environment. The imprecise parameters of the inventory model can be represented as a trapezoidal fuzzy number. A trapezoidal fuzzy number A is defined by membership function as

$$\mu_{\tilde{A}}(x) = \frac{x - a_1}{a_2 - a_1}, \quad a_1 \le x \le a_2,$$
$$= 1, \quad a_2 \le x \le a_3,$$
$$= \frac{x - a_4}{a_3 - a_4}, \quad a_3 \le x \le a_4,$$
$$= 0, \quad \text{otherwise.}$$

where a_1, a_2, a_3 and a_4 are the lower limit, lower mode, upper mode and upper limit respectively.

The fuzzy inventory model incorporates graded mean integration representation in the model. The membership function of $\tilde{A}$ is represented as

$$\mu_{\tilde{A}}(x) = l(x), \quad x < m,$$
$$= 1, \quad m \le x \le n,$$
$$= u(x), \quad x > n$$

where $l(x)$ is continuous from right and strictly increasing for $x < m$ and $u(x)$ is continuous from left and strictly decreasing for $x > n$.

If l^{-1} and u^{-1} are the inverse functions of l and u respectively then graded mean representation of $\tilde{A}$ is

$$Q(\tilde{A}) = \frac{1}{\int_0^1 \gamma d\gamma} \int_0^1 \left(\frac{l^{-1}(\gamma) + u^{-1}(\gamma)}{2} \right) \gamma d\gamma$$

The graded mean representation for a trapezoidal fuzzy number in inventory $\tilde{A} = (a_1, a_2, a_3, a_4)$ is obtained as

$$Q(\tilde{A}) = \frac{a_1 + 2a_2 + 2a_3 + a_4}{6}$$

The model is converted into fuzzy one by considering costs A_{si} and A_i as fuzzy parameters.

These parameters are represented by trapezoidal fuzzy numbers as

$$A_{si} = (A_{si} - a_1, A_{si} - a_2, A_{si} + a_3, A_{si} + a_4),$$
$$A_i = (A_i - b_1, A_i - b_2, A_i + b_3, A_i + b_4)$$

where $A_{si} - a_l > 0$ and $A_i - b_l > 0$

Here a_l, b_l, $_{l=1,2,3,4}$ are arbitrary positive numbers which satisfy $a_1 > a_2$, $a_3 < a_4$, $b_1 > b_2$, $b_3 < b_4$.

The total fuzzy cost for the entire model is derived as

$$\tilde{E}\{TC(Q_{ij}^*)\} = \sum_{i=1}^{n} D_i P_{ij} + \tilde{A}_i \left(\frac{I}{2} + \frac{(1-\beta_i) D_i}{Q_{ij}} \right) + R_i \left(\frac{h_{ri} + h_{si}}{2} \right) + \frac{\tilde{A}_{si} m_i (1-\beta_i) D_i}{n_i Q_{ij}^*} + \frac{Q_{ij}^* I P_{ij}}{2} \tag{17}$$

Now fuzzy arithmetic operations are used to evaluate the total cost.

Assume that $\tilde{A}_1 = (a_{11}, a_{12}, a_{13}, a_{14})$ and $\tilde{A}_2 = (a_{21}, a_{22}, a_{23}, a_{24})$ are two trapezoidal fuzzy numbers. The arithmetic operations done as per fuzzy rule are described in Appendix I.
By using the fuzzy arithmetic operations the value for the fuzzy total cost for the system is obtained as

$$\tilde{E}\{TC(Q_{ij}^*)\} = [\,\tilde{E}\{TC_1(Q_{ij}^*)\}, \tilde{E}\{TC_2(Q_{ij}^*)\}, \tilde{E}\{TC_3(Q_{ij}^*)\}, \tilde{E}\{TC_4(Q_{ij}^*)\}\,] \qquad (18)$$

where

$$\tilde{E}\{TC_1(Q_{ij}^*)\} =$$
$$D_i P_{ij} + \left(\frac{I}{2} + \frac{(1-\beta_i)D_i}{Q_{ij}}\right)(A_i - b_1) + R_i\left(\frac{h_{ri} + h_{si}}{2}\right) + \frac{m_i(1-\beta_i)D_i}{n_i Q_{ij}}(A_{si} - a_1) + \frac{Q_{ij} IP_{ij}}{2},$$

$$\tilde{E}\{TC_2(Q_{ij}^*)\} =$$
$$D_i P_{ij} + \left(\frac{I}{2} + \frac{(1-\beta_i)D_i}{Q_{ij}}\right)(A_i - b_2) + R_i\left(\frac{h_{ri} + h_{si}}{2}\right) + \frac{m_i(1-\beta_i)D_i}{n_i Q_{ij}}(A_{si} - a_2) + \frac{Q_{ij} IP_{ij}}{2},$$

$$\tilde{E}\{TC_3(Q_{ij}^*)\} =$$
$$D_i P_{ij} + \left(\frac{I}{2} + \frac{(1-\beta_i)D_i}{Q_{ij}}\right)(A_i + b_3) + R_i\left(\frac{h_{ri} + h_{si}}{2}\right) + \frac{m_i(1-\beta_i)D_i}{n_i Q_{ij}}(A_{si} + a_3) + \frac{Q_{ij} IP_{ij}}{2},$$

$$\tilde{E}\{TC_4(Q_{ij}^*)\} =$$
$$D_i P_{ij} + \left(\frac{I}{2} + \frac{(1-\beta_i)D_i}{Q_{ij}}\right)(A_i + b_4) + R_i\left(\frac{h_{ri} + h_{si}}{2}\right) + \frac{m_i(1-\beta_i)D_i}{n_i Q_{ij}}(A_{si} + a_4) + \frac{Q_{ij} IP_{ij}}{2}.$$

The graded mean representation of trapezoidal fuzzy number is used to achieve the defuzzified value for the total cost for the system as

$$M(\tilde{Q}_{ij}) = [\,\tilde{E}\{TC_1(Q_{ij}^*)\} + 2\,\tilde{E}\{TC_2(Q_{ij}^*)\} + 2\,\tilde{E}\{TC_3(Q_{ij}^*)\} + \tilde{E}\{TC_4(Q_{ij}^*)\}\,]/6 \qquad (19)$$

Thus, the total cost of all brands of items is obtained as

$$M(\tilde{Q}_{ij}) =$$
$$\sum_{i=1}^{n} \frac{1}{6}\left[D_i P_{ij} + \left(\frac{I}{2} + \frac{(1-\beta_i)D_{ij}}{Q_{ij}}\right)(A_i - b_1) + R_i\left(\frac{h_{ri} + h_{si}}{2}\right) + \frac{m_i(1-\beta_i)D_i}{n_i Q_{ij}}(A_{si} - a_1) + \frac{Q_{ij} IP_{ij}}{2}\right] +$$
$$\frac{2}{6}\left[D_i P_{ij} + \left(\frac{I}{2} + \frac{(1-\beta_i)D_{ij}}{Q_{ij}}\right)(A_i - b_2) + R_i\left(\frac{h_{ri} + h_{si}}{2}\right) + \frac{m_i(1-\beta_i)D_i}{n_i Q_{ij}}(A_{si} - a_2) + \frac{Q_{ij} IP_{ij}}{2}\right]$$
$$+\frac{2}{6}\left[D_i P_{ij} + \left(\frac{I}{2} + \frac{(1-\beta_i)D_{ij}}{Q_{ij}}\right)(A_i + b_3) + R_i\left(\frac{h_{ri} + h_{si}}{2}\right) + \frac{m_i(1-\beta_i)D_i}{n_i Q_{ij}}(A_{si} + a_3) + \frac{Q_{ij} IP_{ij}}{2}\right]$$
$$+\frac{1}{6}\left[D_i P_{ij} + \left(\frac{I}{2} + \frac{(1-\beta_i)D_{ij}}{Q_{ij}}\right)(A_i + b_4) + R_i\left(\frac{h_{ri} + h_{si}}{2}\right) + \frac{m_i(1-\beta_i)D_i}{n_i Q_{ij}}(A_{si} + a_4) + \frac{Q_{ij} IP_{ij}}{2}\right] \qquad (20)$$

The optimum order quantity for i^{th} item is determined by differentiating equation (20) partially with respect to Q_{ij} and equating it to zero. Thus,

$$Q_{ij}^{*} = \Bigg\{ \frac{1}{6IP_{ij}}\left[2(1-\beta_i)D_i(A_i-b_1)+\frac{2m_i(1-\beta_i)D_i(A_{si}-a_1)}{n_i}\right]$$
$$+\frac{2}{6IP_{ij}}\left[(1-\beta_i)D_i(A_i-b_2)+\frac{m_i(1-\beta_i)D_i(A_{si}-a_2)}{n_i}\right]$$
$$+\frac{2}{6IP_{ij}}\left[(1-\beta_i)D_i(A_i+b_3)+\frac{m_i(1-\beta_i)D_i(A_{si}+a_3)}{n_i}\right]$$
$$+\frac{1}{6IP_{ij}}\left[2(1-\beta_i)D_i(A_i+b_4)+\frac{2m_i(1-\beta_i)D_i(A_{si}+a_4)}{n_i}\right]\Bigg\}^{1/2} \quad (21)$$

The optimal value of the total cost for the system for all types of items can be obtained by substituting the value of Q_{ij}^{*} in equation (20).

6. Algorithm to find the optimum order quantity

An algorithm for the inventory model has been suggested to compute the optimum order quantity. It is that order quantity for the new items which minimizes the total system cost per unit time for i[th] type product. The following procedure is followed to obtain the optimum order quantity

(i) Find Q_{i3}^{*}.

(ii) If $Q_{i3}^{*} \geq b_2$, then Q_{i3}^{*} is the optimum order quantity.

(iii) Else if $b_1 \leq Q_{i3}^{*} < b_2$, then find Q_{i2}^{*}.

(iv) If $b_1 \leq Q_{i2}^{*} < b_2$ then find $E\{TC(Q_{i2}^{*})\}$ and $E\{TC(b_2)\}$.

(v) If $E\{TC(Q_{i2}^{*})\} < E\{TC(b_2)\}$, then Q_{i2}^{*} is the optimum order quantity else b_2 is the optimum order quantity.

(vi) If $Q_{i2}^{*} < b_1$, then find Q_{i1}^{*}.

(vii) For $Q_{i1}^{*} < b_1$, find $E\{TC(Q_{i1}^{*})\}$ and $E\{TC(b_1)\}$.

(viii) If $E\{TC(Q_{i1}^{*})\} < E\{TC(b_1)\}$, then Q_{i1}^{*} is the optimum order quantity, else b_1 is the optimum order quantity.

7. Numerical Illustration

The analytical results established for the concerned inventory model has been verified numerically using software Matlab 6.5. The effect of various parameters on the optimal order quantity and the total cost is examined for the crisp and fuzzy inventory models. The model is constructed for two brands of items i. e. i=2. The values of other parameters for first type of item are taken as $b_0 = 0$, $b_1 = 300$, $b_2 = 800$, I=0.02, $P_{11} = Rs.10.00$, $P_{12} = Rs.9.25$, $P_{13} = Rs.8.75$, $D_1 = 400$ units, $m_1 = 4$, $n_1 = 3$, β_1=0.84, h_{r1}=Rs. 2 per unit per unit time, h_{s1}=Rs. 4 per unit per unit time, R_1=40 units, while that for another type of item are $P_{21} = Rs.20.00$, $P_{22} = Rs.13.25$, $P_{23} = Rs.10.75$, $D_2 = 600$ units, $m_2 = 6$, $n_2 = 5$, β_2=0.84, h_{r2}=Rs. 5 per unit per unit time, h_{s2}=Rs. 8/unit/unit time, R_2=50 units. Table 1 presents the effect of A_{si} and A_i on Q_{ij}^{*} and $E\{TC(Q_{ij}^{*})\}$ for the crisp mathematical model. Table 2 displays the variation of optimum order quantity and the total cost with $\tilde{A}_{si}$ and $\tilde{A}_i$ for the fuzzy mathematical model.

It is noticed that the optimum order quantity as well as the total cost increases with the increase in the setup cost for recovery process per setup as shown in fig. 3 (a) and 3 (b) for the model with multiple items and two price breaks. It reveals that when the setup cost for recovery per setup is high, it is profitable to order more new items than to recover the older items as it costs lesser. It is also found that as the carrying cost increases, there is decrease in the optimum order quantity but total cost increases. As the amount invested in inventory increases, it would increase the total cost incurred. In order to cut the cost in such a scenario, it is advisable to order lesser new items and this would consequently reduce the inventory as well as the cost incurred. The increase in the ordering cost for new items per order results in the increase in the optimum order quantity as well as in the total cost; this pattern is visible in fig. 4 (a) and 4 (b). This is due to the fact that as the ordering cost for new items per order increases it is beneficial to order more items in an order than to increase the number of order placed. The increase in the ordering cost for new items per order has a direct impact on the total system cost. So, the total cost increases with the increase in the ordering cost of the new items.

The model with fuzzy ordering cost for new items and setup cost for recovery process is also examined. The variation of total cost with $\tilde{A}_{si}$ and $\tilde{A}_i$ is shown in fig. 5 (a) and 5 (b), respectively. It is found that the results of the crisp model and the fuzzy model go hand in hand with each other and they follow the same trend. This shows that the fuzzy logic can be easily used to tackle complex inventory scenario including the multiple items and price breaks.

8. Conclusion

The investigation carried on the inventory model incorporates multiple items with two price breaks. The optimum order quantity and the optimum total cost derived for the crisp as well as fuzzy models may be helpful to the decision makers in determining the quantity to be purchased and the quantity to be recovered in order to minimize the total cost incurred. It is evident that the inventory level must be maintained low when the carrying cost is high. The inventory model may be extended to multiple price breaks. The merits of the present model are

- The model proves to be beneficial for decision makers dealing with different types of commodities together.
- The optimal quantity to be procured may be decided when different discounts are offered for two different range of price.
- The profit can be maximized by reducing the set-up cost and the ordering cost.
- The fuzzy model proves to be beneficial for the decision makers operating with imprecise variables.

A_{si} (Rs.)	A_i (Rs.)	I=0.02					I=0.04				
		Q^*_{1j} (units)	Q^*_{2j} (units)	$E\{TC(Q_{1j})\}$ (Rs.)	$E\{TC(Q_{2j})\}$ (Rs.)	Total Cost (Rs.)	Q^*_{1j} (units)	Q^*_{2j} (units)	$E\{TC(Q_{1j})\}$ (Rs.)	$E\{TC(Q_{2j})\}$ (Rs.)	Total cost (Rs.)
40	300	494	502	3914.50	8411.10	12325.60	350	355	3955.40	8469.20	12424.60
80	300	530	536	3921.10	8419.80	12340.90	375	378	3964.80	8481.70	12446.50
120	300	564	567	3927.40	8428.30	12355.70	399	401	3973.60	8493.60	12467.20
160	300	596	597	3933.30	8436.20	12369.50	421	422	3981.80	8504.80	12486.60
200	300	625	626	3938.80	8443.80	12382.60	442	443	3989.80	8515.40	12505.20
100	280	535	538	3921.70	8420.50	12342.20	378	381	3965.50	8482.40	12447.90
100	320	560	565	3926.80	8427.80	12354.60	396	399	3972.90	84930	12465.90
100	360	584	590	3931.70	8434.90	12366.60	413	417	3980.10	8503.20	12483.30
100	400	607	614	3936.40	8441.70	12378.10	429	434	3986.90	8513.00	12499.90
100	440	629	637	3940.90	8448.20	12389.10	445	450	3993.60	8522.50	12516.10

Table 1 The effect for varying values of set-up cost for recovery process and ordering cost of new item on Q^*_{ij} and $E\{TC(Q^*_{ij})\}$

$\widetilde{A}_{si}$ (Set of trapezoidal fuzzy number)	$Q(\widetilde{A}_{si})$	I=0.02					I=0.04				
		Q_{1j}^* (units)	Q_{2j}^* (units)	$\widetilde{E}\{TC(Q_{1j}^*)\}$ (Rs.)	$\widetilde{E}\{TC(Q_{2j}^*)\}$ (Rs.)	Total Cost (Rs.)	Q_{1j}^* (units)	Q_{2j}^* (units)	$\widetilde{E}\{TC(Q_{1j}^*)\}$ (Rs.)	$\widetilde{E}\{TC(Q_{2j}^*)\}$ (Rs.)	Total Cost (Rs.)
(70,78,104.109)	90.5	440	444	3924.90	8425.20	12350.10	311	314	3970.10	8489.10	12459.20
(75,84,108,113)	95.3	444	447	3925.70	8426.20	12351.90	313	316	3971.20	8490.60	12461.80
(80,88,110,117)	98.8	446	450	3926.20	8427.00	12353.20	316	318	3972.00	8491.70	12463.70
(88,90,113,122)	102.6	449	453	3926.80	8427.70	12354.50	318	320	3972.80	8492.80	12465.60
(90,94,126,130)	110	454	457	3928.00	8429.30	12357.30	321	323	3974.40	8495.00	12469.40
$\widetilde{A}_i$	$Q(\widetilde{A}_i)$										
(240,262,312,324)	285.3	439	442	3924.50	8424.50	12349.00	310	313	3969.50	8488.00	12457.60
(260,274,320,335)	297.2	446	448	3926.00	8426.70	12352.70	315	317	3971.70	8491.20	12462.90
(265,280,335,340)	305.8	450	453	3927.60	8428.30	12355.90	318	320	3973.30	8493.60	12466.90
(270,285,342,355)	313.2	454	457	3928.10	8429.70	12357.80	321	323	3974.70	8495.60	12470.30
(282,294,350,362)	322	458	462	3929.20	8431.30	12360.50	324	327	3976.30	8497.90	12474.20

Table 2. The variation of optimum order quantity and the total cost with $\widetilde{A}_{si}$ and $\widetilde{A}_i$ for the fuzzy model.

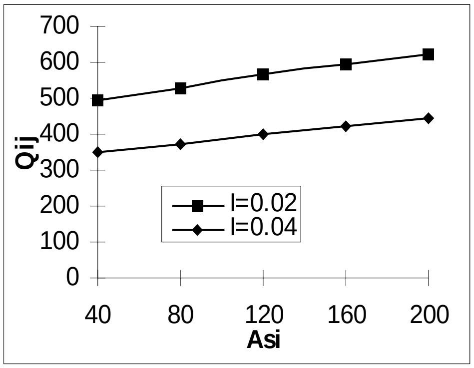


Fig. 3 (a)

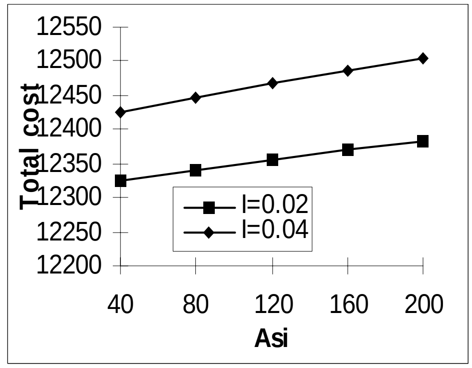


Fig. 3 (b)

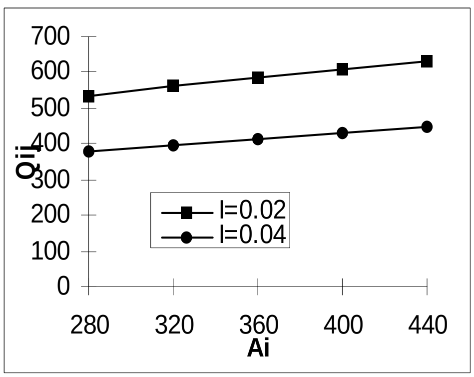


Fig. 4 (a)

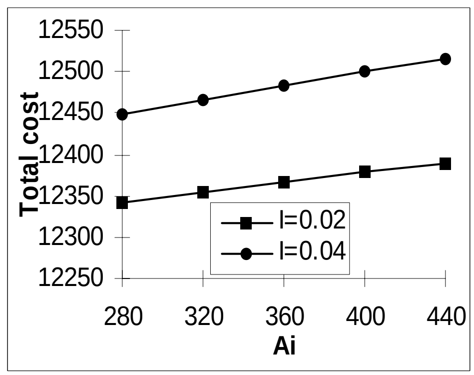


Fig. 4 (b)

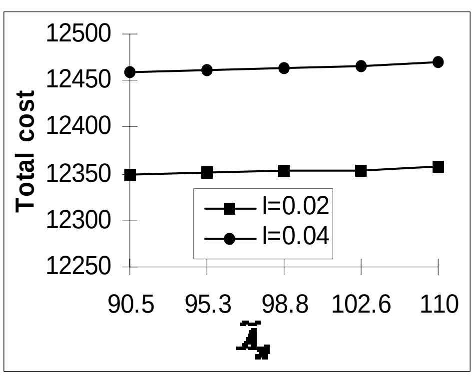


Fig. 5 (a)

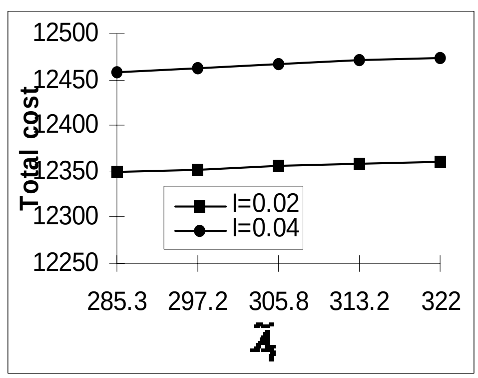


Fig. 5 (b)

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Appendix I

The fuzzy arithmetic operations:

(i) Addition

$$\tilde{A}_1 + \tilde{A}_2 = (a_{11} + a_{21}, a_{12} + a_{22}, a_{13} + a_{23}, a_{14} + a_{24})$$

(ii) Subtraction

$$\tilde{A}_1 - \tilde{A}_2 = (a_{11} - a_{24}, a_{12} - a_{23}, a_{13} - a_{22}, a_{14} - a_{21})$$

(iii) Multiplication

$$\tilde{A}_1 * \tilde{A}_2 = (a_{11}a_{21}, a_{12}a_{22}, a_{13}a_{23}, a_{14}a_{24})$$

(iv) Division

$$\frac{\tilde{A}_1}{\tilde{A}_2} = \left(\frac{a_{11}}{a_{24}}, \frac{a_{12}}{a_{23}}, \frac{a_{13}}{a_{22}}, \frac{a_{14}}{a_{21}}\right)$$

(v) Scalar multiplication

Let $k \geq 0$ be a real number then

$$k\tilde{A}_1 = (ka_{11}, ka_{12}, ka_{13}, ka_{14})$$