Mathematics Today Vol.28(June-Dec-2012)43-49
ISSN 0976-3228

Ferrofluid Lubrication equation for non-isotropic porous squeeze film bearing with slip velocity

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Abstract

The aim of this paper is to derive general ferrofluid lubrication equation for non-isotropic permeable porous squeeze film bearing with slip velocity from momentum and continuity equations.Various special cases are also deduced.

KeywordsFerrofluid, Lubrication, Squeeze film bearing, Slip velocity

AMS 76D08

1. Introduction

Many investigators [1-3] in their analyses used porous bearing with conventional lubricants. Some simplified their analysis [4-6] by using Morgan-Cameron approximation [7] to avoid series solutions of bearing characteristics.

Ferrofluids or magnetic fluids [8] are stable colloidal suspensions containing fine ferromagnetic particles dispersing in liquid carriers, in which a surfactant is added to generate a coating layer preventing the flocculation of the particles. Ferrofluids can experience magnetic body forces depend upon the magnetization of ferromagnetic particles when an externally magnetic field is applied. Owing to these features, ferrofluids have been applied in many areas of industrial engineering and applied science, such as the medicine treatment for drug targeting by Rosensweig [8], the flow sensors by Popa*et. al.*[9].

With the invention of ferrofluids, studies [10-12] were made with ferrofluid-based porous bearings using Neuringer-Rosensweig model for the lubricant flow. Beavers and Joseph [13], Beavers *et. al.* [14] and Sparrow *et. al.*[15] proved analytically and experimentally that the assumption of no slip at the porous interface could not hold when the porous matrix was formed with naturally permeable material like foam. Investigations [16-17] were made using the * **Corresponding author**

boundary conditions proposed by Beavers and Joseph [13] and sparrow *et. al*.[15].

In this paper our aim is to deriveferrofluidlubrication equation for non-isotropic porous squeeze film bearing with slip velocity from momentum and continuity equations.

2. Mathematical formulation The configuration of the bearing shown in figure1 consists of two surfaces(or plates)which is separated by film thickness h (*metre*) which is filled with ferrofluid flowing as per Neuringer-Rosensweigs model [8]. The lower plate is made up of naturally permeable material and the upper plate is solid. The upper plate moves normally towards permeable flat lower plate with uniform velocity $\dot{h}= dh/dt$.

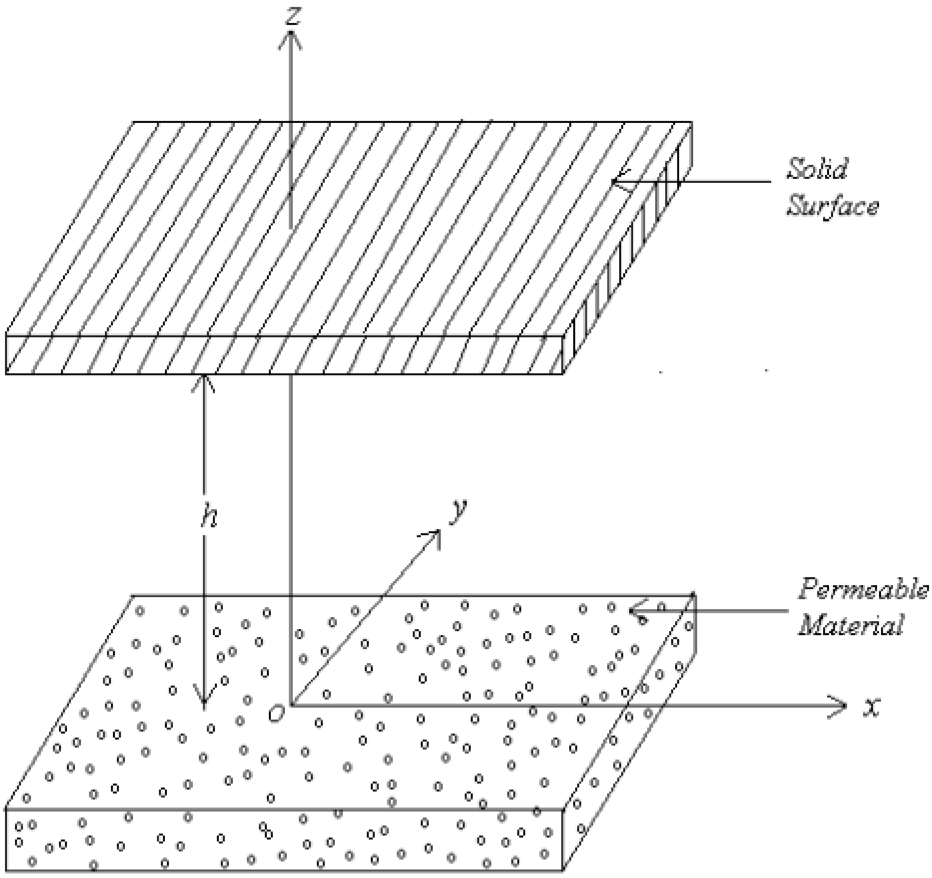

Figure 1.Configuration of the problem.

The following assumptions are made in the derivation (i) there is no variation of pressure across the film, (ii) velocity gradients across the film predominate, (iii) the porous region is non-isotropic permeable, (iv) the flow in the porous region is governed by Darcy's law, (v) pressure and normal velocity components are continuous at the interfaces and (vi) there is a slip velocity at the porous interface.

The basic equations governing the above phenomenon which obtained from Navier-Stroke'sequation are

$$\frac{\partial^2 u}{\partial z^2} = \frac{1}{\eta}\frac{\partial}{\partial x}\left(p - \frac{1}{2}\mu_0\bar{\mu}H^2\right), \tag{2.1}$$

$$\frac{\partial^2 v}{\partial z^2} = \frac{1}{\eta}\frac{\partial}{\partial y}\left(p - \frac{1}{2}\mu_0\bar{\mu}H^2\right), \tag{2.2}$$

where(u,v,w) are film fluid velocities in x, y, z – directions respectively , η is the fluid viscosity , p is pressure of the fluid in the film region,μ_0 is the permeability of free space , $\bar{\mu}$ is the magnetic susceptibility and H is the strength of the magnetic field with its curl and divergence vanishing.

In the porous material having non-isotropic permeability the flow is taken to be Darcian so that

$$\bar{q} = -\frac{1}{\eta}\left[\varphi_x\frac{\partial}{\partial x} + \varphi_y\frac{\partial}{\partial y} + \varphi_z\frac{\partial}{\partial z}\right]\left(P - \frac{1}{2}\mu_0\bar{\mu}H^2\right) \tag{2.3}$$

$$= \bar{u} + \bar{v} + \bar{w}, \tag{2.4}$$

where

$$\bar{u} = -\frac{\varphi_x}{\eta}\frac{\partial}{\partial x}\left(P - \frac{1}{2}\mu_0\bar{\mu}H^2\right), \tag{2.5}$$

$$\bar{v} = -\frac{\varphi_y}{\eta}\frac{\partial}{\partial y}\left(P - \frac{1}{2}\mu_0\bar{\mu}H^2\right), \tag{2.6}$$

$$\bar{w} = -\frac{\varphi_z}{\eta}\frac{\partial}{\partial z}\left(P - \frac{1}{2}\mu_0\bar{\mu}H^2\right), \tag{2.7}$$

where$\bar{q}$ is the Darcian velocity,φ_x ,φ_y ,φ_z are the permeabilities in x,y and z- directions respectively , P is the pressure in porous region.

At the permeable boundary, from continuity consideration

$$P(x,y,z)|_{z=0} = p(x,y). \tag{2.8}$$

Therefore, equation (2.3) becomes

$$\bar{q}_0 = |\bar{q}|_{z=0} = -\frac{1}{\eta}\left[\varphi_x\frac{\partial}{\partial x}\left(p - \frac{1}{2}\mu_0\bar{\mu}H^2\right) + \varphi_y\frac{\partial}{\partial y}\left(p - \frac{1}{2}\mu_0\bar{\mu}H^2\right) + \varphi_z\frac{\partial}{\partial z}\left(P - \frac{1}{2}\mu_0\bar{\mu}H^2\right)|_{z=0}\right] \tag{2.9}$$

$$= \bar{u}_0 + \bar{v}_0 + \bar{w}_0. \tag{2.10}$$

In the film region the equation of continuity is expressed by

$$\frac{\partial}{\partial x}\ (\rho\int_0^h u\ \mathrm{dz}) + \frac{\partial}{\partial y}\ (\ \rho\int_0^h v\ \mathrm{dz}) + (w_h - w_0) + \int_0^h \frac{\partial\rho}{\partial t}\,dz = 0, \tag{2.11}$$

whereρ is the mean density of the fluid which is taken to be independent of *z* coordinate.

Since w_0 denotes the *z* componentof the film fluid velocity at *z* = 0, therefore

$$w_0 = \overline{w}_0 = -\frac{\varphi_z}{\eta}\frac{\partial}{\partial z}\left(P - \frac{1}{2}\mu_0\bar{\mu}H^2\right)|_{z=0}. \tag{2.12}$$

Using slip boundary condition [15] at *z* = 0 ,

$$u = \frac{1}{s_1}\frac{\partial u}{\partial z} \text{ when } z = 0 \text{ and } u = 0 \text{ when} z = h, \tag{2.13}$$

$$v = \frac{1}{s_2}\frac{\partial v}{\partial z} \text{ when} z = 0 \text{ and } v = 0 \text{ when } z = h, \tag{2.14}$$

where

$$\frac{1}{s_1} = \frac{\sqrt{\eta_x\varphi_x}}{5}, \frac{1}{s_2} = \frac{\sqrt{\eta_y\varphi_y}}{5};$$

are slip parameters,$\eta_x$and η_y being the porosities in the *x* and *y* directions respectively.

Integrating equation (2.1) with respect to *z*, we obtain

$$\frac{\partial u}{\partial z} = \frac{1}{\eta}\frac{\partial}{\partial x}(p - \frac{1}{2}\mu_0\bar{\mu}H^2)z + A\ , \tag{2.15}$$

where *A* is a constant of integration.

Again, integrating equation (2.15) with respect to *z*, we obtain

$$u = \frac{1}{\eta}\frac{\partial}{\partial x}(p - \frac{1}{2}\mu_0\bar{\mu}H^2)\frac{z^2}{2} + Az + B, \tag{2.16}$$

where *B* is a constant of integration.

Using first boundary condition of equation (2.13), equation (2.16) yields

$$\frac{1}{s_1}\frac{\partial u}{\partial z}|_{z=0} = B. \tag{2.17}$$

Again, using second boundary condition of equation (2.13), equation (2.16) yields

$$A = -\frac{1}{h}\left[\frac{1}{\eta}\frac{\partial}{\partial x}\ (p - \frac{1}{2}\mu_0\bar{\mu}H^2)\frac{h^2}{2} + \frac{1}{s_1}\frac{\partial u}{\partial z}|_{z=0}\right]. \tag{2.18}$$

Using (2.17) and (2.18), equation (2.16)becomes

$$u = \frac{1}{\eta}\left(\frac{z^2}{2} - \frac{hz}{2}\right)\frac{\partial}{\partial x}\left(p - \frac{1}{2}\mu_0\bar{\mu}H^2\right) + \frac{1}{s_1}\left(1 - \frac{z}{h}\right)\frac{\partial u}{\partial z}\Big|_{z=0}. \tag{2.19}$$

Similarly, using boundary conditions (2.14), equation (2.2)yields

$$v = \frac{1}{\eta}\left(\frac{z^2}{2} - \frac{hz}{2}\right)\frac{\partial}{\partial y}\left(p - \frac{1}{2}\mu_0\bar{\mu}H^2\right) + \frac{1}{s_2}\left(1 - \frac{z}{h}\right)\frac{\partial v}{\partial z}\Big|_{z=0}. \tag{2.20}$$

Substituting the values of *u* and *v* from equation (2.19) and (2.20) in equation (2.11) , we obtain linear partial differential equation of second order in two variables as

$$\frac{\partial}{\partial x}\left[-\frac{h^3}{12\eta}\frac{\partial}{\partial x}\left(p - \frac{1}{2}\mu_0\bar{\mu}H^2\right) + \frac{h}{2s_1}\frac{\partial u}{\partial z}\Big|_{z=0}\right] + \frac{\partial}{\partial y}\left[-\frac{h^3}{12\eta}\frac{\partial}{\partial y}\left(p - \frac{1}{2}\mu_0\bar{\mu}H^2\right) + \frac{h}{2s_2}\frac{\partial v}{\partial z}\Big|_{z=0}\right] + \dot{h} + \frac{\varphi_z}{\eta}\frac{\partial}{\partial z}\left(P - \frac{1}{2}\mu_0\bar{\mu}H^2\right)\Big|_{z=0} + \frac{1}{\rho}\int_0^h \frac{\partial\rho}{\partial t}dz = 0. \tag{2.21}$$

This is the desired ferrofluid lubrication equation for non-isotropic porous squeeze film bearing with slip velocity. This equation is of general nature and can be reduced to the following simpler forms.

3. Discussion of Special Cases

(1) Setting $s_1, s_2 \to \infty$ in Equation (2.21), we obtain the no slip version corresponding to the present case as

$$\frac{\partial}{\partial x}\left[-\frac{h^3}{12\eta}\frac{\partial}{\partial x}\left(p - \frac{1}{2}\mu_0\bar{\mu}H^2\right)\right] + \frac{\partial}{\partial y}\left[-\frac{h^3}{12\eta}\frac{\partial}{\partial y}\left(p - \frac{1}{2}\mu_0\bar{\mu}H^2\right)\right] + \dot{h} + \frac{\varphi_z}{\eta}\frac{\partial}{\partial z}\left(P - \frac{1}{2}\mu_0\bar{\mu}H^2\right)\Big|_{z=0} + \frac{1}{\rho}\int_0^h \frac{\partial\rho}{\partial t}dz = 0.$$

(2) Setting $H = 0$ in Equation (2.21), we obtain the equation for the conventional lubricant considering slip velocity and anisotropic permeability as

$$\frac{\partial}{\partial x}\left[-\frac{h^3}{12\eta}\frac{\partial p}{\partial x} + \frac{h}{2s_1}\frac{\partial u}{\partial z}\Big|_{z=0}\right] + \frac{\partial}{\partial y}\left[-\frac{h^3}{12\eta}\frac{\partial p}{\partial y} + \frac{h}{2s_2}\frac{\partial v}{\partial z}\Big|_{z=0}\right] + \dot{h} + \frac{\varphi_z}{\eta}\frac{\partial P}{\partial z}\Big|_{z=0} + \frac{1}{\rho}\int_0^h \frac{\partial\rho}{\partial t}dz = 0.$$

4. Conclusions

Equation (2.21) is the most general form, using Neuringer-Rosensweig model, for squeeze film bearing for non-isotropic porosity and slip velocity. It can be used to study the performance of the squeeze film bearing of various shapes.Also, it can be further developed using other equations for porous matrix.

Acknowledgement The authors are grateful to the Referee for their valuable suggestions.

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NOMENCLATURE

$\dot{h}$ Squeeze velocity (m/s)

u, v, w Film fluid velocities (m/s)

x, y, z Coordinates (m)

p Pressure of the fluid in the film region (kg/ms^2)

η Fluid viscosity (kg/ms)

μ_0 Free space permeability ($kg\,m/s^2A^2$)

$\bar{\mu}$ Magnetic Susceptibility

H Strength of magnetic field (A/m)

$\bar{q}$ Darcian velocity (m/s)

$\varphi_x, \varphi_y, \varphi_z$ Permeabilities in x, y and z directions (m^2)

P Pressure of the fluid in the porous region (kg/ms^2)

ρ Mean density of the fluid (kg/m^3)

η_x, η_y Porosities in x and y directions