A Study on the Performance of an Infinitely Long Rough Slider Bearing In The Presence of a Magnetic Fluid Lubricant

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Abstract

It is well known fact that the standard deviation associated with the roughness plays a crucial role in improving the performance of an infinitely long hydrodynamic slider bearing. In order to compensate this negative effect of standard deviation it was deemed proper to study and analyze the effect of the standard deviation on the performance of an infinitely long slider bearing taking the magnetic fluid as the lubricant. The random roughness is characterized by stochastic random variable. The associated stochastically averaged Reynolds’ equation is solved with suitable boundary conditions. Expressions for dimensionless pressure, load carrying capacity and friction are obtained. Computed values are displayed graphically. It is observed that a magnetic fluid lubricant improves the performance of the bearing system. It is noticed that the effect of the standard deviation is considerably adverse. Further, the friction decreases at the bearing surfaces while there is an almost negligible increase in the friction at the runner plate. It is revealed that the load carrying capacity increases and the friction decreases at the bearing surfaces and the runner plate with respect to the increasing values of the aspect ratio. This investigation suggests that the outlet film thickness ratio plays a better role as compared to that of the aspect ratio. Besides, this article makes it clear that the adverse effect of the standard deviation can be compensated up to considerably large extent by the positive effect of magnetic fluid lubricant choosing a proper combination of the aspect ratio and the ratio of the outlet film thickness to the length of the bearing.

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Key words: 
Slider bearing, magnetic fluid, roughness, pressure, load carrying capacity, friction.

NOMENCLATURE

h     Fluid film thickness at any point (mm)
h₁     Maximum film thickness (mm)
h₂     Minimum film thickness (mm)
H     Magnetic field
L     Length of the bearing (mm)
h₂/L     The ratio of the outlet film thickness to the length of the bearing
H     Magnitude of H
k     Suitably chosen constant to suit the dimensions of both the sides of the strength of the magnetic field.
m     Aspect ratio
u     Velocity in the X - direction
p     Lubricant pressure (N/mm²)
P     Dimensionless pressure
w     Load carrying capacity (N)
W     Non-dimensional load carrying capacity
φ     Inclination of the magnetic field.
µ₀     Magnetic susceptibility
µ     Free space permeability
µ     Lubricant viscosity (N.s/mm²)
µ*     Dimensionless magnetization parameter
τ     Shear stress (N/mm²)
τ       Dimensionless shear stress
F     Frictional force (N/mm²)
F     Dimensionless frictional force
F₀     Frictional force (at moving plate)
F₁     Frictional force (at fixed plate)
σ     Standard deviation
α     Variance
ε     Measure of symmetry
σ*     Standard deviation in dimensionless form

1. INTRODUCTION

The fundamental aspect in a hydrodynamic slider bearing is the formation of a converging wedge of the lubricant. The hydrodynamic slider may be constructed to provide this converge wedge in a number of ways. Purday [1] established that the shape of the wedge was not that important and all that mattered was in the aspect ratio. The analysis of hydrodynamic lubrication of a non-porous slider is a classical one for instance; one can have a look at Pinkus and Sternlicht [2]. The infinitely long slider bearing is the idealization of single sector shaped pad of a hydrodynamic thrust bearing. Such a bearing consists of a fixed pad or pivoted pad and a moving pad (slider) which may be plane stepped, curved or composite shaped. These types of bearings are designed to support axial loads. Such bearings are widely used in hydroelectric generators, gas turbines and other equipments. In a landmark paper Lord Rayleigh [3] analyzed the film profile for a one dimensional slider bearing carrying maximum
load and concluded that it is step shaped with step height ratio being 1.866 and the lower step being 28.2% of the slider length. Exact solutions of Reynolds’ equation for thrust bearing with various simple film geometries are presented in a number of books and research papers (Cameron [4], Gross et. al. [5], Archibald [6], Charnes and Seibel [7], Basu et. al [8], Hamrock [9] Majumdar [10]). Prakash and Vij [11] investigated the performance of an inclined plane infinite slider bearing with an impermeable slider and a porous faced stator backed by a solid wall. Bhat [12] modified the analysis of the above-mentioned paper to discuss the performance of a porous composite slider bearing. Bhat and Patel [13] carried out the analysis in another direction by involving the squeeze film which caused increased load carrying capacity without changing the friction. All the above analyses assumed the bearing surfaces to be smooth. However, the bearing surfaces after having some run-in and wear develop roughness. In order to study and analyze the effect of surface roughness several investigators have proposed a stochastic approach to mathematically model the random character of the roughness (Tzeng and Seibel [14], Christensen and Tonder [15, 16, 17]). Subsequently, this approach of Christensen and Tonder [15, 16, 17] was used in various investigations to discuss the effect of surface roughness on the performance of the bearing systems (Ting [18], Prakash and Tiwari [19], Prajapati [20], Guha [21], Gupta and Deheri [22]).

The above said investigations dealt with the conventional lubricants. Agrawal [23] considered the configuration of Prakash and Vij [11] with a magnetic fluid lubricant and found that the performance was better than the one with conventional lubricant. Bhat and Deheri [24] extended the analysis of Agrawal [23] by investigating the magnetic fluid based porous composite slider bearing. Here it was observed that magnetic fluid lubricant increased the load carrying capacity, unaltered the friction and shifted the centre of pressure towards the inlet. Deheri and Patel [25] discussed the performance of a porous slider bearing with squeeze film formed by a magnetic fluid. Deheri, Andharia and Patel [26] studied the effect of transverse surface roughness in a magnetic fluid based squeeze film slider bearing. Here it was shown that the negative effect of transverse surface roughness could be minimized up to some extent by the positive effect of a magnetic fluid lubricant. The discussions contained in Deheri, Andharia and Patel [26] and Andharia, Gupta and Deheri [27] indicated that the standard deviation associated with the characterization of roughness played a crucial role in infinitely long bearing while the effect of variance and skewness was not that sharp. Thus, it has been proposed to analyze the performance of a magnetic fluid based transversely rough infinitely long bearing wherein, the standard deviation plays the dominant role over mean and skewness.

2. ANALYSIS

The configuration of the bearing which is infinite in Z – direction is shown in Figure (i).
The non-porous slider moves with a uniform velocity \( u \) in the \( X \)-direction. The length of the bearing is \( L \). The bearing surfaces are assumed to be transversely rough. The geometry of the local film thickness can be thought of as consisting of two parts (Christensen and Tonder [15, 16, 17])

\[
h(x) = \bar{h}(x) + h_s(x)
\]

where \( \bar{h}(x) \) is the mean film thickness and \( h_s(x) \) is the deviation from the mean film thickness characterizing the random roughness of the bearing surfaces. \( h_s(x) \) is considered to be stochastic in nature and governed by the probability density function \( f(h_s) \), \(-c \leq h_s \leq c \) where \( c \) is the maximum deviation from the mean film thickness. The mean \( \alpha \), the standard deviation \( \sigma \) and the parameter \( \epsilon \) which is the measure of symmetry of random variable \( h_s \), are defined by relationships

\[
\begin{align*}
\alpha &= E(h_s) \\
\sigma^2 &= E[(h_s - \alpha)^2] \\
\epsilon &= E[(h_s - \alpha)^3]
\end{align*}
\]

and

\[
f(h_s) = \begin{cases} 
\frac{35}{32c^7}(c^2 - h_s^2)^3 & -c \leq h_s \leq c \\
0 & \text{elsewhere}
\end{cases}
\]

The magnetic field is oblique to the stator as in Agrawal [23]. Magnitude of the applied magnetic field is taken be a function of \( x \). We assume that the magnetic field has components of the form

\[
\mathbf{H} = H(x)(\cos\phi, 0, \sin\phi); \quad \phi = \phi(x, z)
\]

while \( H^2 \) satisfies the condition that it becomes zero at the interface of the bearing and the atmosphere which means

\[
H^2(x) = 0 \text{ at } x = 0 \text{ and } x = L.
\]

Hence

\[
\nabla \times \mathbf{H} = 0
\]

in the present case leads to the equation for the inclination of magnetic field \( \phi \) as

\[
\cot\phi \frac{\partial \phi}{\partial x} + \frac{\partial \phi}{\partial z} = -\frac{1}{H} \frac{dH}{dx}
\]

(2.1)

Following Prajapati [28] it is considered that

\[
H^2 = kL^2 \sin\left(\frac{\pi x}{L}\right)
\]

where \( L \) is the length of the bearing along \( X \)-axis. Hence the direction \( \phi \) of the magnetic field is given by the \( C \) elimination of

\[
\csc\left(\frac{\pi x}{L}\right) = C^2 \sin\phi
\]

and

\[
-C - \frac{\pi x}{L} = \int \frac{d\phi}{C\sin^2\phi - 1}
\]

(2.2)
With the usual assumptions of magnetohydrodynamic lubrication together with the assumption that the self field created by the magnetization of the fluid is neglected, the associated Reynolds’ equation for the pressure distribution turns out to be (Prajapati [28], Basu et. al. [8])

$$\frac{d}{dx} \left( p - \frac{\mu_0 \mu H^2}{2} \right) = 6\mu_0 u \frac{Th_2 - \lambda h_2}{h_2^2 T^3 + 3\sigma^2 h_2 T^2} \tag{2.3}$$

where

$$T = \left\{ 1 + m \left( 1 - \frac{x}{L} \right) \right\},$$

$$\lambda > 1$$ and is given by

$$\lambda = \frac{\tan^{-1} \left\{ \frac{\sqrt{3}\sigma mh_2}{h_2^2 (1 + m) + 3\sigma^2} \right\}}{h_2 \ln \left\{ \frac{(1 + m)\sqrt{h_2^2 + 3\sigma^2}}{\sqrt{h_2^2 (1 + m)^2 + 3\sigma^2}} \right\}},$$

$$\mu_0$$ is the magnetic susceptibility, $$\mu$$ is free space permeability, $$\mu$$ is lubricant viscosity and $$m$$ is the aspect ratio.

The associated boundary conditions are

$$p = 0$$ at $$x = 0$$ and $$x = L$$.

Solving Equation (2.3) in view of the above boundary conditions one gets the pressure distribution as

$$p = \frac{\mu_0 \mu k L^2 \sin \left( \frac{\pi x}{L} \right)}{2} + \frac{2\sqrt{3}Lu\mu}{m\sigma h_2} \{ U + V \} \tag{2.4}$$

where

$$U = \tan^{-1} \left\{ \frac{\sqrt{3}\sigma h_2^2 (1 + m) - Th_2}{h_2^2 (1 + m)T + 3\sigma^2} \right\}$$

and

$$V = \tan^{-1} \left\{ \frac{\sqrt{3}\sigma mh_2}{h_2^2 (1 + m) + 3\sigma^2} \right\} \ln \frac{T\sqrt{h_2^2 (1 + m)^2 + 3\sigma^2}}{(1 + m)\sqrt{h_2^4 T^2 + 3\sigma^2}} \ln \frac{(1 + m)\sqrt{h_2^2 + 3\sigma^2}}{\sqrt{h_2^2 (1 + m)^2 + 3\sigma^2}}.$$
Introducing the dimensionless quantities

\[ m = \frac{h_2 - h_1}{L} \quad X = \frac{x}{L} \quad P = \frac{h_2^2 p}{\mu u L^2} \quad \bar{h}_2 = \frac{h_2}{L} \quad \mu^* = \frac{h_2^2 k \mu_0}{\mu u} \]

\[ Y = \frac{y}{h} \quad \sigma^* = \frac{\sigma}{L} \quad \bar{T} = \left[1 + m(1 - X)\right] \quad J = \frac{\sqrt{3} \sigma^*}{h_2} \]

one obtains the expression for pressure distribution in dimensionless form as

\[ P = \frac{\mu^* \sin(\pi X)}{2} + \frac{2 \sqrt{3} h_2^2}{\sigma^* m} \left(\bar{U} + \bar{V}\right) \] (2.5)

where

\[ \bar{U} = \tan^{-1} \left(\frac{(-mX) / J}{(1 + (1/J)^2(1 + m)\bar{T})}\right) \]

and

\[ \bar{V} = \tan^{-1} \left(\frac{m / J}{(1 + (1/J)^2(1 + m))}\right) \ln \left(\frac{\bar{T}(1 + m)^2 + (J)^2}{(1 + m)\sqrt{T^2 + (J)^2}}\right) \]

\[ \ln \left(\frac{(1 + m)(1 + (1/J)^2)}{\sqrt{(1 + m)^2 + (J)^2}}\right) \]

The non-dimensional load carrying capacity of the bearing is given by

\[ W = \frac{h_2^3 w \pi}{\mu u L^4} = \frac{1}{\pi} \int_0^1 P dX \] (2.6)

Therefore, the dimensionless load carrying capacity of the bearing can be expressed as

\[ W = \mu^* + \frac{2 \sqrt{3} \pi h_2^2}{m \sigma^*} \left[\left(\frac{m^2 - 1}{m}\right) \tan^{-1}(A) - \frac{J}{2m} \ln(C) - \frac{J \tan^{-1} A}{\ln[C(1 + m)]}\right] \] (2.7)

where

\[ A = \frac{m / J}{1 + \frac{(m + 1)}{J^2}} \quad \text{and} \quad C = \frac{\sqrt{1 + J^2}}{\sqrt{(1 + m)^2 + J^2}}. \]

The frictional force \( \bar{F} \) per unit width of the lower plane of the moving plate is obtained as

\[ \bar{F} = \int_0^1 \tau dX \]

where

\[ \tau = \left(\frac{h_2}{u \mu}\right) \sigma \]

is non-dimensional shearing stress, while

\[ \tau = \frac{dp}{dx} \left(\frac{y - h}{2}\right) + \frac{\mu u}{h} \]
On simplifications this yields,

\[ \tau = \frac{dP}{dX} \frac{Y - \frac{1}{2}}{h^2} + \frac{1}{T} \] (2.8)

At \( Y = 0 \) (at moving plate), one finds that

\[ \tau = -\mu^* \frac{\pi T \cos(\pi X)}{4h^2} - \frac{T}{2h^2} \left[ \left( \frac{2h_2^3}{\sigma^2} \right) \left( \frac{1}{1 + (\bar{T}/J)^2} \right) - 6\sqrt{3}m^* \left( \frac{G}{T^2 + J^2} \right) \right] + \frac{1}{T} \] (2.9)

where

\[ G = \frac{\tan^{-1} \left( \frac{mJ}{T^2 + (1 + m)} \right)}{\ln \left( \frac{(1 + m)\sqrt{1 + J^2}}{(1 + m)^2 + J^2} \right)} \]

Thus, in non-dimensional form the frictional force assumes the form

\[ F_0 = -\frac{\mu^*}{2h_2\pi^2} + \frac{3}{2m} \ln \left( \frac{J^2 + 1}{J^2 + (1 + m)^2} \right) + \frac{3\sqrt{3}\sigma^* G}{2h_2} \ln \left( \frac{J^2 + (1 + m)^2}{J^2 + 1} \right) + \frac{\ln(1 + m)}{m} \] (2.10)

Further, at \( Y = 1 \) (at fixed plate), one obtains that

\[ \tau = \frac{\mu^* \pi T \cos(\pi X)}{4h^2} + \frac{T}{2h^2} \left[ \left( \frac{2h_2^3}{\sigma^2} \right) \left( \frac{1}{1 + (\bar{T}/J)^2} \right) - 6\sqrt{3}m^* \left( \frac{G}{T^2 + J^2} \right) \right] + \frac{1}{T} \] (2.11)

Lastly, in dimensionless form the frictional force at the fixed plate comes out to be

\[ F_1 = -\frac{\mu^*}{2h_2\pi^2} - \frac{3}{2m} \ln \left( \frac{J^2 + 1}{J^2 + (1 + m)^2} \right) - \frac{3\sqrt{3}\sigma^* G}{2h_2} \ln \left( \frac{J^2 + (1 + m)^2}{J^2 + 1} \right) + \frac{\ln(1 + m)}{m} \] (2.12)

3. RESULTS AND DISCUSSION

The expressions for non-dimensional pressure distribution and load carrying capacity are given by Equations (2.5) and (2.7) respectively. Further, the friction at the bearing plate and the runner plate are determined from equations (2.10) and (2.12). Taking the magnetization parameter and roughness parameter to be zero this study reduces to the discussion of Basu et al. [8]. A comparison with the conventional lubricants indicates that the non-dimensional pressure increases by

\[ \frac{\mu^* \sin(\pi X)}{2} \]

and the load carrying capacity gets increased by \( \mu^* \).

Furthermore, the friction at the moving plate gets reduced and at the fixed plate it increases by

\[ \frac{\mu^*}{2\pi h_2} \].
A cursory glance at the results shows that the non-dimensional load carrying capacity in the present case is approximately four times more than the case of the magnetic field wherein, the magnitude is taken as

$$H^2 = kx(L - x).$$

It can be easily observed that the roughness has an adverse effect in general, wherein, the standard deviation plays a dominant role in reducing the load carrying capacity.

The distribution of non-dimensional load carrying capacity with respect to the magnetization parameter for different values of standard deviation, aspect ratio and the ratio $h_2/L$ presented in Figures (1) – (3) respectively, makes it clear that the load carrying capacity increases sharply. However, the increase in the case of the aspect ratio is relatively sharp. The effect of the standard deviation is given in Figures (4) and (5). It is easily seen that the standard deviation has a considerable negative effect in the sense that the load carrying capacity decreases sharply, the decrease being more at the initial stages. Figure (6) suggests that the combined effect of the aspect ratio and the ratio $h_2/L$ is significantly positive.

The variation of frictional force at the moving plate with respect to magnetization parameter for different values of the standard deviation, the aspect ratio and the outlet film thickness ratio $h_2/L$ is depicted in Figures (7) – (9). It is indicated that the friction decreases considerably with respect to the magnetization parameter $\mu^*$. Also, the standard deviation causes reduced friction. Besides, the effect of $h_2/L$ with respect to the aspect ratio is almost negligible. However, it is revealed that there is a nominal increase due to magnetization at the fixed plate but the standard deviation decreases the friction. The effect of outlet film thickness ratio with respect to the aspect ratio is almost similar to the corresponding case at the bearing plate.

Some of these Figures tend to suggest that negative effect of the standard deviation can be compensated up to a large extent by the positive effect of the magnetization by choosing suitably the aspect ratio and the outlet film thickness ratio.

4. CONCLUSIONS

Thus, the magnetic fluid lubricant improves the performance of the bearing system; even the bearing can support a load in the absence of flow. This study makes it clear that from bearing’s life period point of view the roughness must be accounted for while designing the bearing system even if, there is the presence of magnetic fluid lubricant. This discussion underlines that equally crucial is the role of the aspect ratio particularly when large value of outlet film thickness ratio is involved. Lastly, this investigation proceeds to offer an additional degree of freedom for designing the bearing system, in the form of the magnitude of the magnetic field.

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REFERENCES


Fig. 1 Variation of load carrying capacity with respect to $\mu^*$ and $\sigma^*$.

Fig. 2 Variation of load carrying capacity with respect to $\mu^*$ and $m$.

Fig. 3 Variation of load carrying capacity with respect to $\mu^*$ and $h_2/L$. 
Fig. 4 Variation of load carrying capacity with respect to $\sigma^*$ and $m$.

Fig. 5 Variation of load carrying capacity with respect to $\sigma^*$ and $h^2/L$.

Fig. 6 Variation of load carrying capacity with respect to $m$ and $h^2/L$. 
Fig. 7 Variation of frictional force at moving plate with respect to $\mu^*$ and $\sigma^*$.

Fig. 8 Variation of frictional force at moving plate with respect to $\mu^*$ and $m$.

Fig. 9 Variation of frictional force at moving plate with respect to $\mu^*$ and $h_2/L$. 