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Role of Modified Chaplygin Gas in Little Rip Cosmological Models in Kaluza-Klein Theory of Gravitation

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Abstract

In this paper, we studied the influence of time dependent parameters $\omega(t)$ and $\Lambda(t)$ in the equation of state of Chaplygin gas upon the occurrence of Little Rip/Pseudo Rip in Kaluza-Klein type cosmological model of the universe.

Keywords: Cosmology, higher dimensional space time, Little Rip, Pseudo Rip, Chaplygin gas.

1 Introduction

Current observations [Perlmutter et al. (1999), Riess (1998), Kowalski et al. (2008), Hicken et al. (2009)] strongly suggest that the universe is dominated by a negative-pressure component, dubbed dark energy. This component can be characterized by an equation of state parameter ω which is simply the ratio of the pressure to the density: $\omega = \frac{p}{\rho}$; for example, a cosmological constant corresponds to $\omega = -1$. While it is often assumed that $\omega \geq -1$ in accordance with the weak energy condition, it has been known since long time[Caldwell (2002)] that the observations are consistent with $\omega < -1$, which corresponds to a dark energy density that increases with time t and scale factor R. If the density increases monotonically in the future, then the universe

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can undergo a future singularity, called the ”big rip”, for which $\rho \to \infty$ and $R \to \infty$ at a finite time. Shortly before this singularity is reached, bound structures are disintegrated by the expansion [Caldwell et al. (2003)].

Note, however, that a dark energy component with a monotonically increasing density that is unbounded from above does not lead inevitably to a future singularity, although it does ultimately lead to the dissolution of all bound structures. Such models, dubbed ”Little Rip” (LR) models, were first examined in detail by Frampton et al. (2011), who derived the boundary between big rip and little rip models in terms of $\rho(R)$. Properties of little rip models were further investigated in Ref. [Frampton et al. (2012)].

On the other hand, the Pseudo-Rip (PR) dissociates the bound structures which are held together by a binding force at or below a particular threshold, and hence it is possible that only some bound structures are dissociated while the others are not dissociated [Frampton, et al. (2012)].

Recent observations of the luminosity of type Ia supernovae indicate [Bachall et al. (1999), Perlmutter et al. (1999)] an accelerated expansion of the universe and lead to the search for a new type of matter which violate the strong energy condition $\rho + 3p < 0$. The matter content responsible for such a condition to be satisfied at a certain stage of evolution of the universe is referred to as dark energy. There are different candidates to play the role of the dark energy. The type of dark energy represented by a scalar field is often called quintessence. The transition from a universe filled with matter to an exponentially expanding universe does not necessarily require the presence of the scalar field as the only alternative. In particular one can try another alternative by using an exotic type of fluid - the so-called Chaplygin gas which obeys an equation of state like $p = -\frac{B}{\rho}$ [Kamenshchik et al. (2001)] where p and ρ are respectively pressure and energy density and B is a positive constant. Subsequently the above equation was modified to the form $p = -\frac{B}{\rho^{\alpha}}$ with $0 \leq \alpha \leq 1$. This model gives the cosmological evolution from an initial dust like matter to an asymptotic cosmological constant with an epoch that can be seen as a mixture of a cosmological constant and a fluid obeying an equation of state $p = \alpha\rho$. This generalized model has been studied previously [Gorini et al. (2003); Alam et al. (2003); Bento et al. (2002)].

Bervik et al. (2012) investigated the dark energy models with an inhomogeneous equation of state leading to the LR cosmology in the far future and shown that the LR cosmology is caused exponentially, determined by the parameters ω and Λ in the frame work of general theory of relativity.

With the motivation of this work, in this paper we study the dark energy model with time dependent parameters ω and Λ in the equation of state of Chaplygin gas in which LR/PR behavior is encountered in the frame work of Kaluza-Klein theory of gravitation. Estimates for the time required for disintegration of gravitational bound systems are given.

2 Model and Field Equations

We consider flat Kaluza-Klein type cosmological model of the form

$$ds^2 = dt^2 - R^2(t)[dr^2 + r^2(d\theta^2 + sin^2\theta d\phi^2) + d\psi^2], \tag{1}$$

where $R(t)$ is the scale factor.
Einstein field equations are given as

$$G_{ij} = R_{ij} - \frac{1}{2}g_{ij}R = -8\pi G T_{ij}, \tag{2}$$

the energy momentum tensor for matter source is given by

$$T_{ij} = (p + \rho)\mu_i\mu_j - pg_{ij},$$

where ρ and p are energy density and pressure.
Einstein field Eqs.(2) for the model Eq.(1) are given by

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{\chi^2\rho}{6}, \tag{3}$$

$$\frac{\ddot{R}}{R} + \left(\frac{\dot{R}}{R}\right)^2 = -\frac{\chi^2 p}{3}, \tag{4}$$

where the overhead dot denotes differentiation with respect to time t and $\chi^2 = 8\pi G$ is the gravitational constant.
From Eq.(3) we get

$$\frac{6}{\chi^2}H^2 = \rho, \tag{5}$$

where $H = \frac{\dot{R}}{R}$ the Hubble parameter.
From the field equations Eq.(3) and Eq.(4), the energy conservation law is given by

$$\dot{\rho} + 4H(p+\rho) = 0. \tag{6}$$

We consider a general modified Chaplygin gas obeying an equation of state

$$p = \omega(t)\rho - \frac{\Lambda(t)}{\rho^{\alpha}}, \tag{7}$$

where p is pressure and $\omega(t)$, $\Lambda(t)$ are time-dependent parameters.
We assume the various form of LR model in 5D cosmology with a given Hubble parameter H:
Case (a): $H(t) = H_0 e^{\lambda t}$ and Case (b): $H(t) = H_0 e^{Ce^{\lambda t}}$.
In both cases when $t \to \infty$, $H \to \infty$.

2.1 Case (a): $H(t) = H_0 e^{\lambda t}, \quad H_0 > 0, \quad \lambda > 0$

With present time $t = 0$, we get $H = H_0$, so that H_0 becomes the present-time Hubble parameter.

In this case after differentiating Eq.(5) with respect to t we get

$$\dot{\rho} = \frac{12\lambda}{\chi^2} H^2, \tag{8}$$

By using equation of state Eq.(7); Eq.(5) and Eq.(8) in the conservation in Eq.(7), we get

$$\frac{12\lambda}{\chi^2} H^2 + 4H \left[\frac{6}{\chi^2}(1+\omega(t))H^2 + \Lambda(t) \left(\frac{6}{\chi^2} \right)^{-\alpha} H^{-2\alpha} \right] = 0. \tag{9}$$

After solving equation Eq.(9) for the value of $\omega(t)$ we have

$$\omega(t) = \frac{-\lambda}{2H} + \left(\frac{6}{\chi^2} \right)^{-(\alpha+1)} \Lambda(t) H^{-2(\alpha+1)} - 1. \tag{10}$$

Let us investigate various cases:

We assume that $\Lambda(t) = constant$ i. e. $\Lambda(t) = \Lambda_0$ then $\omega(t) \to -1$ for $\alpha > 0$ asymptotically from below, in the far future. Thus for an ideal fluid obeying EOS Eq.(7) and Eq.(10) the LR scenario is found.
If we consider $\omega(t)$ does not depend on time, $\omega(t) = constant$ i.e. $\omega(t) = \omega_0$, then from Eq.(9) for the value of $\Lambda(t)$ can be expressed as

$$\Lambda(t) = \left(\frac{6}{\chi^2} \right)^{\alpha+1} H^{2(\alpha+1)} \left[\frac{\lambda}{2H} + (1+\omega_0) \right]. \tag{11}$$

As $\omega_0 > 0$ in order to correspond to a dark fluid, we obtain $\Lambda(t) \to \infty$ in the far future. The LR behavior in this case is caused by the parameter ω_0.

2.2 Case (b): $H(t) = H_0 e^{Ce^{\lambda t}}$, H_0, C, $\lambda > 0$

For this case from the Eq.(5) we get

$$\rho = \frac{6}{\chi^2} H_0^2 e^{2Ce^{\lambda t}}, \tag{12}$$

After differentiating Eq.(12) with respect to t, we get

$$\dot{\rho} = \frac{12\lambda}{\chi^2} H^2 ln \frac{H}{H_0}. \tag{13}$$

Now, Eq.(6) becomes

$$\frac{12\lambda}{\chi^2} H^2 ln \frac{H}{H_0} + 4H \left[\frac{6}{\chi^2}(1 + \omega(t))H^2 - \Lambda(t) \left(\frac{6}{\chi^2} \right)^{-\alpha} H^{-2\alpha} \right] = 0. \tag{14}$$

To solve this equation we assume that the parameter $\Lambda(t)$ in terms of Hubble parameters of the form

$$\Lambda(t) = \gamma H^{2(\alpha+1)}, \tag{15}$$

where γ is a constant.

By using Eq.(15) into Eq.(14) we obtain $\omega(t)$ as

$$\omega(t) = \frac{-\lambda}{2H} ln \frac{H}{H_0} - \gamma \left(\frac{6}{\chi^2} \right)^{-(\alpha+1)} - 1. \tag{16}$$

Now consider parameter $\omega(t)$ in the form

$$\omega(t) = -\frac{\delta}{H^2} - 1, \tag{17}$$

where δ is a positive constant.

We obtain from Eq.(14)

$$\Lambda(t) = \left(\frac{6}{\chi^2} \right)^{\alpha} H^{2\alpha} \left[\frac{3\lambda}{\chi^2} H ln \frac{H}{H_0} - \frac{6\delta}{\chi^2} \right]. \tag{18}$$

2.3 Pseudo-Rip Model

In this section we consider the Pseudo-Rip model with different behavior of H:

$$H(t) = H_0 - H_1 e^{-\lambda t}, \tag{19}$$

where H_0, H_1 and λ are positive constants.
We assume that $H_0 > H_1$ when $t > 0$.
From the Eq.(5), we obtain

$$\rho = \frac{6}{\chi^2}(H_0 - H_1 e^{-\lambda t})^2. \tag{20}$$

On differentiating Eq.(20) with respect to t we obtain

$$\dot{\rho} = \frac{12\lambda}{\chi^2} H(H - H_0). \tag{21}$$

With the help of above equations the energy conservation equation Eq.(6) can be expressed as

$$\frac{12\lambda}{\chi^2} H(H - H_0) + 4H\left[\frac{6}{\chi^2}[1+\omega(t)]H^2 - \Lambda(t)\left(\frac{6}{\chi^2}\right)^{-\alpha} H^{-2\alpha}\right] = 0. \tag{22}$$

First, if we assume $\Lambda(t) = constant = \Lambda_0$, then from Eq.(22) gives

$$\omega(t) = \frac{-\lambda(H - H_0)}{2H^2} + \Lambda_0\left(\frac{6}{\chi^2}\right)^{-(\alpha+1)} H^{-2(\alpha+1)} - 1, \tag{23}$$

which shows that the PR behavior is determined by the parameter Λ_0.
Now we consider again $\Lambda(t) = \gamma H^{2(\alpha+1)}$ in Eq.(22) we get

$$\omega(t) = \frac{-\lambda(H - H_0)}{2H^2} + \gamma\left(\frac{6}{\chi^2}\right)^{-(\alpha+1)} - 1. \tag{24}$$

Solving Eq.(22) for $\Lambda(t)$ by taking $\omega(t) = constant = \omega_0$, we find

$$\Lambda(t) = \left(\frac{6}{\chi^2}\right)^{\alpha} H^{2\alpha}\left[\frac{3\lambda}{\chi^2}(H - H_0) + \frac{6}{\chi^2}(1+\omega_0)H^2\right]. \tag{25}$$

For Eq.(17), we obtain $\Lambda(t)$ as

$$\Lambda(t) = \left(\frac{6}{\chi^2}\right)^{\alpha} H^{2\alpha}\left[\frac{3\lambda}{\chi^2}(H - H_0) - \frac{6\delta}{\chi^2}\right]. \tag{26}$$

Thus, we have presented the appearance of LR and PR from the modified Chaplygin gas obeying an equation of state of the form Eq.(7).
In the following section we investigate the influence from the parameters ω and Λ in the equation of state for Chaplygin gas upon the time needed for disintegration in the LR/PR models.

2.4 The Inertial Force in LR/PR Cosmology

A particle with mass m at a given point will be subject to an inertial force is given by Frampton et al. (2012) of the form

$$F_{in} = ml\frac{\ddot{R}}{R} = ml(\dot{H} + H^2), \tag{27}$$

Let us assume that two particles are bounded by a constant gravitational force F_0. If $F_{in} > 0$ and $F_{in} > F_0$, the particles become unbounded. It is convenient to define the dimensionless parameter in Kaluza-Klein theory of gravittaion as

$$\bar{F_{in}} = \frac{2\rho(R) + \rho^{'}(R)R}{\rho_0} = 12\frac{\ddot{R}}{R\rho_0}, \tag{28}$$

where ρ_0 is the dark energy density at present.
By substituting the value of H and it's derivative from case (a) in the Eq.(27) we get the inertial force

$$F_{in} = ml(\lambda H_0 e^{\lambda t} + H_0^2 e^{2\lambda t}). \tag{29}$$

Hence, at time $t \to +\infty$ the inertial force $F_{in} \to +\infty$. This characterizes the LR model under certain conditions.
Now, we investigate the influence from the parameters ω and Λ in the equation of state for Chaplygin gas upon the Little Rip time $t = t_{LR}$ for the case (a): $H = H_0 e^{\lambda t}$.
In the equation of state Eq.(6) we take

$$\omega(t) = \omega_0,$$

$$\omega_0 < -1, \tag{30}$$

$$\Lambda(t) = \Lambda_0,$$

where ω_0 and Λ_0 are constants.
Now we consider the value of ω_0 and Λ_0 in the following form:

$$\omega_0 = -1 - \frac{\lambda}{3} \;\; and \;\; \Lambda_0 = \frac{\lambda}{\chi^2}\left(\frac{6}{\chi^2}\right)^{\alpha}. \tag{31}$$

With the help of above value the energy conservation law Eq.(6) becomes

$$\frac{2\lambda}{\chi^2} H_0^{2(\alpha+1)} e^{2(\alpha+1)\lambda t} - \frac{3\lambda}{\chi^2} H_0^{2\alpha+1} e^{(2\alpha+1)\lambda t} + \frac{\lambda}{\chi^2} = 0. \tag{32}$$

We solve the Eq.(32) for the disintegration time $t = t_{LR}$, one finds

$$t_{LR} = \frac{1}{\lambda} ln \left[\frac{1}{H_0} \right]. \tag{33}$$

For the PR model with the help of Eq.(19), the inertial force Eq.(27) can be written as

$$F_{in} = ml[\lambda H_1 e^{-\lambda t} + (H_0 - H_1 e^{-\lambda t})^2]. \tag{34}$$

In this case the inertial force is $F_{in} \to ml H_0^2$ when $t \to +\infty$.
Now we consider

$$\omega_0 = -1 + \frac{\lambda H_0}{2}, \quad \Lambda_0 = \frac{3\lambda}{\chi^2} \left(\frac{6}{\chi^2} \right)^{\alpha}, \tag{35}$$

then the energy conservation Eq.(6) reduces in the form of

$$\frac{3\lambda H_0}{\chi^2} H^{2(\alpha+1)} + \frac{3\lambda}{\chi^2} H^{2\alpha+1} - \frac{3\lambda H_0}{\chi^2} H^{2\alpha} - \frac{3\lambda}{\chi^2} = 0. \tag{36}$$

We solve the Eq.(36) for the disintegration time $t = t_{PR}$, to get

$$t_{PR} = \frac{-1}{\lambda} ln \left[\frac{H_0 - 1}{H_1} \right]. \tag{37}$$

It is thus possible, from the relations Eq.(33) and Eq.(37), to estimate the span of time t_{LR} or t_{PR} needed before the system becomes gravitationally unbound.

3 Conclusion

In this paper, we have presented dark energy models with equation of state of Chaplygin gas in which LR or PR behavior is encountered in far future in the framework Kaluza-Klein theory of gravitation. We have studied the influence of the time dependent parameters ω and Λ in the equation of state (7) upon the occurrence of LR/PR for different values of Hubble parameter H. We have shown that LR cosmology is caused exponentially with cosmological constant Λ and parameter ω and PR behavior is determined by the parameter Λ_0. It is of interest to note that the disintegration of bound structures in the LR/PR models may occur for physically acceptable choice of parameters in the context of Kaluza-Klein theory. It is thus possible from the relations (33) and (37) to estimates the disintegration times in both LR/PR models.

References

[1] Perlmutter, S.J. et al., Measurements of Ω and Λ from 42 High-Redshift Supernovae: *Astrophys. J.* **517**, 565 (1999).

[2] Riess, A.G. et al., Observational Evidence from Supernovae for an Accelerating Universe and a Cosmological Constant: (Supernova Search Team), *Astron. J.* **116**, 1009 (1998).

[3] Kowalski, M., et al., Improved Cosmological Constraints from New, Old, and Combined Supernova Data Sets: *Astrophys. J.* **686**, 749 (2008).

[4] Hicken M., et al., CfA3: 185 Type Ia Supernova Light Curves From The CfA: *Astrophys. J.* **700**, 1097 (2009).

[5] Caldwell, R.R., A Phantom Menace? Cosmological consequences of a dark energy component with super-negative equation of state: *Phys. Lett.* **B545**, 23 (2002).

[6] Caldwell, R.R., Kamionkowski, M. and Weinberg, N.N., Phantom Energy: Dark Energy with $\omega < -1$ Causes a Cosmic Doomsday: *Phys. Rev. Lett.* **91**, 071301 (2003).

[7] Frampton, P.H., Ludwick K.J. and Scherrer, R.J., The Little Rip: *Phys. Rev.* **D84**, 063003 (2011) [arXiv:1106.4996].

[8] Frampton, P.H., Ludwick, K.J., Nojiri, S., Odintsov, S.D. and Scherrer, R.J., Models for little rip dark energy: *Phys. Lett.* **B708**, 204 (2012).

[9] Bachall, N.A., Ostriker, J.P., Perlmutter, S. and Steinhardt, P.J., The Cosmic Triangle: Revealing the State of the Universe: *Science* **284**, 1481 (1999).

[10] Kamenshchik, A., Moschella, U. and Pasquier, V., An alternative to quintessence: *Phys. Lett.* **B511**, 265 (2001).

[11] Gorini, V., Kamenshchik, A. and Moschella, U., Can the Chaplygin gas be a plausible model for dark energy?: *Phys. Rev.* **D67**, 063509 (2003).

[12] Alam, U., Sahni, V., Saini, T.D. and Starobinsky, A.A., Exploring the expanding Universe and dark energy using the statefinder diagnostic: *Mon. Not. Roy. Astron. Soc.* **344**, 1057 (2003).

[13] Bento, M.C., Bertolami, O. and Sen, A.A., Generalized Chaplygin gas, accelerated expansion, and dark-energy-matter unification: *Phys. Rev.* D66, 043507 (2002).

[14] Brevik, I., Obukhov, V.V., Osetrin, K.E. and Timoshkin, A.V., Little Rip Cosmological Models With Time Dependent equation of State: *Mod. Phys. Lett. A* **27**, 1250210 (2012).