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Real Life Application of Fuzzy Logic

Dr.Shraddha Ingale

P. G. Dept. of Mathematics & Research Center,

New Arts, Commerce & Science College, Ahmednagar (Maharashtra)

The word fuzzy has dictionary meaning as blurred, indistinct; unclear, misty, distorted, out of focus, unfocused, lacking definition, low resolution, nebulous; indefinite, vague, hazy, imprecise, inexact.

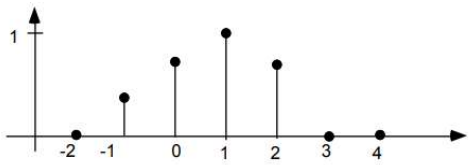
Fuzzy theory consists of fuzzy logic(FL) based on fuzzy set theory. The deep roots of FL has been laid by Plato indicating that there was a third region beyond true or false. Lukasiewicz first proposed a systematic alternative of many valued logic to the bivalued logic. FL is much closer in spirit to human thinking and natural language than the traditional (classical) logic. Fuzzy ideas and FL are so often utilized in our routine life that nobody even pays attention to them. For instance, to answer some questions in certain surveys, most time one could answer with 'Not Very Satisfied' or 'Quite Satisfied', which are also fuzzy or ambiguous answers. FL provides an effective means of capturing the approximate, inexact nature of the real world.

Fuzzy logic was invented by Professor L. A. Zadeh of the University of California at Berkeley in 1965. But was recognized in 1974 by E. H. Mamdani when applied it to control an automatic steam engine. FL has a number of applications in diverse fields such as industrial process control, military, medical diagnosis, finance, risk analysis, environmental science, management, psychology, music, decision making, automatic speech recognition, radionuclide safety etc.

Classical logic or Boolean logic deals with only 0 and 1 i.e. True or False. It's extension is infinite valued FL, accepting that things can be partly true and partly false at the same time. Hence it is a logic of inexact or ambiguous concepts. Fuzzy Logic provides way to calculate with imprecision and vagueness. FL deals with degrees of membership (degrees of truth) and linguistic variables (having values as words or sentences in natural language), modifiers, rules and approximate reasoning.

Fuzzy Sets: One approach for classical or a crisp set A ⊆ X is listing it's members, while another is identifying a crisp set by it's characteristic function $\chi_A : X \rightarrow \{0, 1\}$. Similarly, in fuzzy sets, every member of the set has certain degree of belongingness to the set, which is known as a membership degree and the function assigning this is known as a membership function $\mu_A: X \rightarrow [0,1]$.

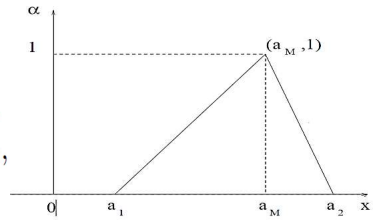
A discrete fuzzy set is represented as a collection of order pairs i.e. $\boldsymbol{A} = \{(x, \mu_A(x))/ x \in X\}$



A discrete membership function for "x is close to 1".

A continuous fuzzy set may be triangular, trapezoidal, bell shaped or part of these.
A triangular fuzzy set is defined as:

$$\mathbf{A} \triangleq \mu_{\mathbf{A}}(x) = \begin{cases} \frac{x-a_1}{a_M-a_1} & \text{for } a_1 \le x \le a_M, \\ \frac{x-a_2}{a_M-a_2} & \text{for } a_M \le x \le a_2, \\ 0 & \text{otherwise,} \end{cases}$$



An alpha cut is subset of $[a_1, a_2]$ defined as:

$$A_\alpha = \{x \mid x \in R, \mu_{\mathcal{A}}(x) \ge \alpha\}, \quad \alpha \in [0, 1]$$

To implement fuzzy logic technique to a real application requires the following three steps:

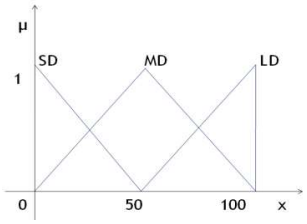
1. **Fuzzification** – convert classical data into Membership Functions (MFs)
2. **Fuzzy Inference Process** – combine membership functions with the control rules (from knowledge of an expert) in If-Then format with and/or connectives to derive the fuzzy output.
3. **Defuzzification** – to convert fuzzy output into crisp value.

Use of fuzzy logic in Washing Machine:

For a very simple model, suppose in order to wash clothes in a machine, the clothes are dirty/greasy. So, input variables are considered as Dirt and Grease which are measured on the scale from 0 to 100. Also let Wash Time to be an output variable having value from 0 to 60 minutes.

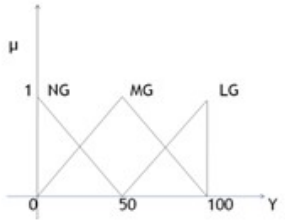
Define membership function for Dirt (SD:Small Dirt, MD:Medium Dirt, LD: Large Dirt) as:

$$\mu_{dirt}(x) = \begin{cases} \mu_{SD}(x) = \frac{50-x}{50} & for\ 0 \le x \le 50 \\ \mu_{MD}(x) = \begin{cases} \frac{x}{50} & for\ 0 \le x \le 50 \\ \frac{100-x}{50} & for\ 50 \le x \le 100 \end{cases} \\ \mu_{LD}(x) = \frac{x-50}{50} & for\ 50 \le x \le 100 \end{cases} \quad \ldots\ldots\ldots(1)$$



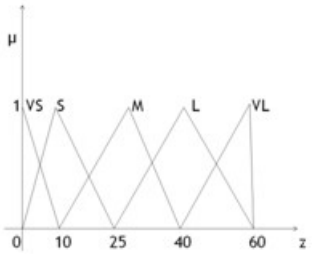
Similarly define the membership function for Grease (NG: No Grease, MG: Medium Grease, LG: Large Grease) also:

$$\mu_{grease}(x) = \begin{cases} \mu_{NG}(x) = \frac{50-x}{50} & for\ 0 \le x \le 50 \\ \mu_{MG}(x) = \begin{cases} \frac{x}{50} & for\ 0 \le x \le 50 \\ \frac{100-x}{50} & for\ 50 \le x \le 100 \end{cases} \\ \mu_{LG}(x) = \frac{x-50}{50} & for\ 50 \le x \le 100 \end{cases} \quad \text{.....(2)}$$



For wash time (VS-Very Small, S-Small, M-Medium, L-Large, VL-Very Large) the fuzzy set is:

$$\mu_{wash\ time}(x) = \begin{cases} \mu_{VS}(x) = \frac{10-x}{10} & for\ 0 \le x \le 10 \\ \mu_{S}(x) = \begin{cases} \frac{x}{10} & for\ 0 \le x \le 10 \\ \frac{25-x}{15} & for 10 \le x \le 25 \end{cases} \\ \mu_{M}(x) = \begin{cases} \frac{x-10}{15} & for\ 10 \le x \le 25 \\ \frac{40-x}{15} & for\ 25 \le x \le 40 \end{cases} \\ \mu_{L}(x) = \begin{cases} \frac{x-25}{15} & for\ 25 \le x \le 40 \\ \frac{60-x}{20} & for\ 40 \le x \le 60 \end{cases} \\ \mu_{VL}(x) = \frac{x-40}{20} & for\ 40 \le x \le 60 \end{cases} \quad \text{.....(3)}$$



Define ***fuzzy rules*** as a relation between Dirt, Grease: (VS, S, M, L, VL are WashTime values)

		Grease		
		NG	**MG**	**LG**
Dirt	**SD**	VS	M	L
	MD	S	M	L
	LD	M	L	VL

The above table represents 9 rules: They can be read as:
If Medium Dirt (MD) and Large Grease (LG) then Wash Time is Large (L) and so on.

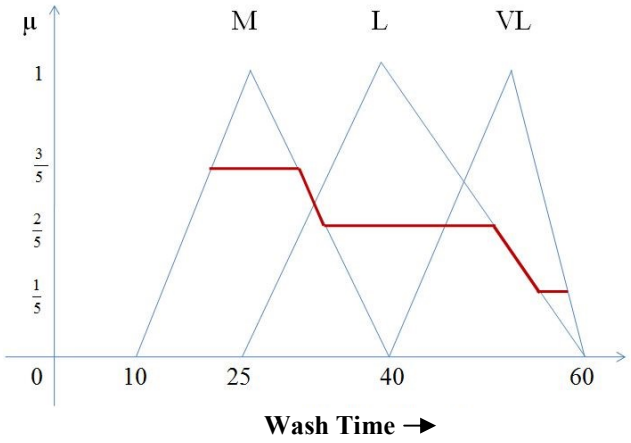
Suppose the sensor reading on machine identifies values for Dirt to be 60 and Grease as 70. Then by using equations (1) and (2), we get : $\mu_{SD}(60) = 0$, $\mu_{MD}(60) = 4/5$, $\mu_{LD}(60) = 1/5$ and $\mu_{NG}(60) = \mu_{MG}(70) = 3/5$, $\mu_{LG}(70) = 2/5$

Form this we take only non-zero values indicating that only four rules defined in above table are applicable. Consider their output,

min{*Dirt (MD), Medium Grease(MG)*} = min {4/5, 3/5} = 3/5 cut of *Washtime (M)*
min{*Dirt (MD), Large Grease(LG)*} = min {4/5, 2/5} = 2/5 cut of *Washtime (L)*
min{*Dirt (LD), Medium Grease(MG)*} = min {1/5, 3/5} = 1/5 cut of *Washtime (L)*
min{*Dirt (LD), Large Grease(LG)*} = min {1/5, 2/5} = 1/5 cut of *Washtime (VL)*

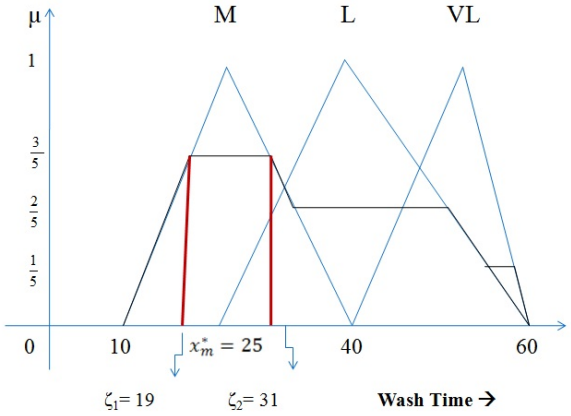
Note that, outcome of second and third rules is Large (Wash Time) having 2/5 and 1/5 cut resp. The rule related to max{2/5, 1/5}= 2/5 is taken into consideration. As a result, only three rules will be fired(activated).

On plot of Wash Time plotting all these cuts we get output as a fuzzy set (bold red border) as below,



Since output Wash Time should be a crisp value, defuzzification procedure is required.

By use of mean of maxima defuzzification method, wash time value corresponding to average value/mean value of maximum alpha cut(as shown in fig.) is taken as a output which is (19+31)/2 = 25 minutes (Wash Time required).



We can make the expert system to be more precise by adding extra input variables like amount of washing powder, water consumption, etc.

There are other methods listed in literature, Center of Gravity, Centroid Method, Averaging Method, Bisector Method. Using Matlab; Fuzzy Logic tool and defuzzification as a centroid method for above example the output is 33.7 minutes. An expert's opinion is taken in to consideration for the choice of resulting value obtained by different defuzzification methods.

Industrial Applications of fuzzy logic and fuzzy sets developed at initial stage are Cement kiln controller in Denmark(1976), Automatic control system of a Train at Sundai in Japan (1987), Water treatment systems(FRUITAX) by Fuji Electric, in finance, Yamaichi Fuzzy Fund (dealing 65 industries, 800 fuzzy rules), Commercial Household products like Shower Head (first consumer product),Washing Machine - first one named as *Asai-go(beloved wife) day fuzzy*, Vacuum Cleaners, Rice Cookers, Refrigerators etc.

Drawbacks of fuzzy logic:

It requires tuning of membership functions, Fuzzy Logic control may not scale well to large or complex problems and it deals with imprecision, and vagueness, but not uncertainty.

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