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Strong Domination and m - splitting in Graphs

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Abstract

Let G be a graph with vertex set V(G) and edge set E(G). Then we say u strongly dominates v (v weakly dominates u) if $deg(u) \geq deg(v)$ and $uv \in E(G)$. A subset D of $V(G)$ is called a strong(weak) dominating set of G if every vertex v in $V(G) - D$ is strongly(weakly) dominated by some $u \in D$. The smallest cardinality of strong (weak) dominating set is called a strong (weak) domination number. In this paper we explore the concept of strong domination number and investigate the same for the graph obtained by m - splitting of the given graph.

Keywords: Domination number, strong domination number, d- balanced graph.

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1 Introduction

We consider finite, undirected, connected graph without loops and multiple edges. The vertex set and edge set of the graph G is denoted by $V(G)$ and $E(G)$ respectively. For any graph theoretic terminology and notations we rely upon Chartrand and Lesniak [4]. We denote the degree of a vertex v in a graph G by $d(v)$. The maximum and minimum degree of the graph G is denoted by $\triangle(G)$ and $\delta(G)$ respectively.

A set $D \subseteq V(G)$ of vertices in the graph G is called a dominating set if every vertex $v \in V(G)$ is either an element of D or is adjacent to an element of D. A dominating set D is a minimal dominating set if no proper subset $D^{'} \subset D$ is a dominating set. The domination number $\gamma(G)$ of G is the minimum cardinality of a minimal dominating set of the graph G. For the better understanding of domination theory and its related concepts we refer to Haynes *et al*. [9] while a detailed bibliography on the concept of domination can be found in Hedetniemi and Laskar [8].

We will give some basic definitions required for the present work.

Definition 1.1. *[2] The m - splitting graph $S'_m(G)$ of a graph G is obtained by adding to each vertex v of G m new vertices, say $v_1, v_2, v_3, \dots, v_m$ such that v_i, $(1 \leq i \leq m)$ is adjacent to each vertex that is adjacent to v in G.*

Definition 1.2. *[12] For graph G and $uv \in E(G)$, we say u strongly dominates v (v weakly dominates u) if $deg(u) \geq deg(v)$.*

Definition 1.3. *[12] A subset D is a strong(weak) dominating set $sd-set(wd-set)$ if every vertex $v \in V(G) - D$ is strongly(weakly) dominated by some u in D. The strong(weak) domination number $\gamma_{st}(G)(\gamma_w(G))$ is the minimum cardinality of a $sd-set(wd-set)$.*

Definition 1.4. *Let $G = (V(G), E(G))$ be a graph and $D \subset V(G)$. Then D is s-full (w-full) if every $u \in D$ strongly (weakly) dominates some $v \in V(G) - D$.*

Definition 1.5. *A graph G is domination balanced $(d - balanced)$ if there exists an sd-set D_1 and a wd-set D_2 such that $D_1 \cap D_2 = \phi$.*

The concepts of strong and weak domination was introduced by Sampathkumar and Pushpa Latha [12]. Many results on the concepts of strong and weak domination have been established by Domke *et al* [6]. Various bounds on the strong domination number are achieved by Rautenbach in [10]. The analogous work have been carried out in Hattingh and Henning [7]. The influence of special vertices on strong domination is well studied by Rautenbach in [11]. Many graph-theoretic concepts like independence, irredundance, packing and domatic numbers are studied in the context of theory of dominating sets by Cockayne and Hedetniemi [5] as well as by Allan and Laskar [1]. For regular graphs $\gamma_{st}(G) = \gamma_w(G) = \gamma(G)$ is obvious as reported by Swaminathan and Thangaraju [13]. Therefore we consider the graphs which are not regular.

2 Main Results

We begin with some propositions required to prove the results in present paper.

Proposition 2.1. *[12] For a graph G of order n, $\gamma \le \gamma_{st} \le n - \triangle(G)$.*

Proposition 2.2. *[3] For a nontrivial path P_n,*

$$\gamma_{st}(P_n) = \left\lceil \frac{n}{3} \right\rceil \text{ and } \gamma_w(P_n) = \begin{cases} \left\lceil \frac{n}{3} \right\rceil & \text{if } n \equiv 1 (mod\ 3) \\ \left\lceil \frac{n}{3} \right\rceil + 1 & \text{otherwise} \end{cases}$$

Proposition 2.3. *[3] For cycle C_n, $\gamma_{st}(C_n) = \gamma_w(C_n) = \gamma(C_n) = \left\lceil \frac{n}{3} \right\rceil$.*

Proposition 2.4. *[12] For a graph G, the following statements are equivalent.*

1. *G is d-balanced.*
2. *There exists an sd-set D which is s-full.*
3. *There exists an wd-set D which is w-full.*

Theorem 2.5. $\gamma[S'_m(P_n)] = \gamma_{st}[S'_m(P_n)] = \begin{cases} \frac{n}{2} & \text{if } n \equiv 0 (mod\ 4) \\ \frac{n+1}{2} & \text{if } n \equiv 1, 3 (mod\ 4) \\ \frac{n}{2} + 1 & \text{if } n \equiv 2 (mod\ 4) \end{cases}$

Proof: let $v_1, v_2, \dots, v_n$ be the vertices of the path P_n. Let v_i^j $(1 \le j \le m)$ be the vertices corresponding to each v_i for $(1 \le i \le n)$ which are added in the path P_n to obtain $S'_m(P_n)$ such that $N(v_i^1) = N(v_i^2) = \dots = N(v_i^m)$. Then $|V[S'_m(P_n)]| = nm + n$. The $d(v_1) = d(v_n) = m + 1$, $d(v_2) = \dots = d(v_{n-1}) = 2m + 2 = \triangle[S'_m(P_n)]$, $d(v_1^j) = d(v_n^j) = 1$ while $d(v_2^j) = d(v_3^j) = \dots d(v_{n-1}^j) = 2$.

We consider following cases to prove the result.

Case(i) $n \equiv 0 (mod\ 4)$
Let $V[S'_m(P_n)] = \{v_1, v_2, v_3, v_4, v_1^j, v_2^j, v_3^j, v_4^j\} \cup \{v_5, v_6, v_7, v_8, v_5^j, v_6^j, v_7^j, v_8^j\} \cup \dots \cup \{v_{n-3}, v_{n-2}, v_{n-1}, v_n, v_{n-3}^j, v_{n-2}^j, v_{n-1}^j, v_n^j\}$. Thus $V[S'_m(P_n)]$ can be expressed as a union of $\frac{n}{4}$ vertex disjoint subsets.

The vertices v_1^j are pendant vertices which are adjacent to v_2 as shown in Figure 2.1. To dominate these pendant vertices v_2 must be in any dominating set. The vertex v_2 dominates v_1, v_1^j, v_3, v_3^j, alongwith itself. Now to dominate the vertices of type v_2^j's we either need to take vertex v_1 or the vertex v_3 in the dominating set. Let us take v_3 in the dominating set as it dominates $2m+2$ vertices while v_1 dominates $m+1$ vertices. Thus v_3 dominates vertices of type v_2^j's as well as v_4^j's. Therefore it is enough to take the vertices v_2 and v_3 to dominate the first vertex disjoint subset. There are $\frac{n}{4}$ such vertex disjoint subsets. Thus minimum $2(\frac{n}{4})$ vertices are essntial to dominate the $S'_m(P_n)$. Hence $|D| \geq \frac{n}{2}$. Then the dominating set $D = \{v_2, v_3, v_6, v_7, v_{10}, v_{11} \ldots, v_{n-2}, v_{n-1}\}$ Now, $N[D] = \{v_1, v_2, \ldots, v_n, v_1^j, v_2^j, \ldots, v_n^j\} = V[S'_m(P_n)]$. Hence D is the required dominating set of minimum cardinality. Also, $d(v_2) = \ldots = d(v_{n-1}) = 2m+2 = \triangle[S'_m(P_n)]$. Therefore D is a strong dominating set with minimum cardinality. Thus,

$$\gamma[S'_m(P_n)] = \gamma_{st}[S'_m(P_n)] = \frac{n}{2}, \text{ for } n \equiv 0\,(mod\ 4)$$

Case(ii): $n \equiv 1, 3\,(mod\ 4)$

We consider following subcases to prove the result.

Subcase(i) $n \equiv 1\,(mod\ 4)$

Let $V[S'_m(P_n)] = \{v_1, v_2, v_3, v_4, v_1^j, v_2^j, v_3^j, v_4^j\} \cup \{v_5, v_6, v_7, v_8, v_5^j, v_6^j, v_7^j, v_8^j\} \cup \ldots \cup \{v_{n-4}, v_{n-3}, v_{n-2}, v_{n-1}, v_{n-4}^j, v_{n-3}^j, v_{n-2}^j, v_{n-1}^j\} \cup \{v_n, v_n^j\}$. Thus $V[S'_m(P_n)]$ can be expressed as a union of $\frac{n}{4}+1$ vertex disjoint subsets.

As discussed in Case(i) minimum $\frac{n-1}{2}$ vertices are required to dominate $\frac{n}{4}$ vertex disjoint subsets. Now, we only need to dominate the last vertex disjoint subset $\{v_n, v_n^j\}$. The vertices in the last vertex disjoint subset are adjacent to a single vertex v_{n-1} of the previous vertex disjoint subset as shown in Figure 2.1. Then alongwith $\frac{n-1}{2}$ number of vertices, it is also essential to take v_{n-1} in the dominating set. Hence $|D| \geq \frac{n-1}{2} + 1 = \frac{n+1}{2}$.

Then the dominating set $D = \{v_2, v_3, v_6, v_7, v_{10}, v_{11} \ldots, v_{n-3}, v_{n-2}, v_{n-1}\}$ Now, $N[D] = \{v_1, v_2, \ldots, v_n, v_1^j, v_2^j, \ldots, v_n^j\} = V[S'_m(P_n)]$. Hence D is the required dominating set of minimum cardinality. Also, $d(v_2) = \ldots = d(v_{n-1}) = 2m+2 = \triangle[S'_m(P_n)]$. Therefore D is a strong dominating set with minimum cardinality. Thus,

$$\gamma[S'_m(P_n)] = \gamma_{st}[S'_m(P_n)] = \frac{n+1}{2}, \text{ for } n \equiv 1\,(mod\ 4)$$

Subcase(ii) $n \equiv 3\,(mod\ 4)$

Let $V[S'_m(P_n)] = \{v_1, v_2, v_3, v_4, v_1^j, v_2^j, v_3^j, v_4^j\} \cup \{v_5, v_6, v_7, v_8, v_5^j, v_6^j, v_7^j, v_8^j\} \cup \ldots \cup \{v_{n-6}, v_{n-5}, v_{n-4}, v_{n-3}, v_{n-6}^j, v_{n-5}^j, v_{n-4}^j, v_{n-3}^j\} \cup \{v_{n-2}, v_{n-1}, v_n, v_{n-2}^j, v_{n-1}^j, v_n^j\}$. Thus $V[S'_m(P_n)]$ can be expressed as a union of $\frac{n}{4}+1$ vertex disjoint subsets.

As discussed in Case(i) minimum $\frac{n-3}{2}$ vertices are required to dominate $\frac{n}{4}$ vertex disjoint subsets. Now, we only need to dominate the last vertex disjoint subset $\{v_{n-2}, v_{n-1}, v_n, v_{n-2}^j, v_{n-1}^j, v_n^j\}$. The vertices v_n^j are pendant vertices which are adjacent to vertex v_{n-1}. Then it is required to take v_{n-1} in the dominating set. Now, the vertex v_{n-1} dominates v_n, v_n^j, v_{n-2} and v_{n-2}^j. The vertices of type v_{n-1}^j's are not dominated by v_{n-1}. These v_{n-1}^j are adjacent to v_{n-2} and v_n. So, to dominate these v_{n-1}^j vertices we can either take v_{n-2} or v_n in the dominating set. Let us take v_{n-2} in the dominating set. Hence to dominate the last vertex disjoint subset minimum two vertices are required. Hence $|D| \geq \frac{n-3}{2} + 2 = \frac{n+1}{2}$.

Then the dominating set $D = \{v_2, v_3, v_6, v_7, v_{10}, v_{11} \ldots, v_{n-4}, v_{n-2}, v_{n-1}\}$ Now, $N[D] = \{v_1, v_2, \ldots, v_n, v_1^j, v_2^j, \ldots, v_n^j\} = V[S'_m(P_n)]$. Hence D is the required dominating set of minimum cardinality. Also, $d(v_2) = \ldots = d(v_{n-4}) = d(v_{n-2}) = d(v_{n-1}) = 2m+2 = \triangle[S'_m(P_n)]$. Therefore D is a strong dominating set with minimum cardinality. Thus,

$$\gamma[S'_m(P_n)] = \gamma_{st}[S'_m(P_n)] = \frac{n+1}{2}, \text{ for } n \equiv 3\,(mod\ 4)$$

Case (iii) $n \equiv 2\,(mod\ 4)$

Let $V[S'_m(P_n)] = \{v_1, v_2, v_3, v_4, v_1^j, v_2^j, v_3^j, v_4^j\} \cup \{v_5, v_6, v_7, v_8, v_5^j, v_6^j, v_7^j, v_8^j\} \cup \ldots \cup \{v_{n-5}, v_{n-4}, v_{n-3}, v_{n-2}, v_{n-5}^j, v_{n-4}^j, v_{n-3}^j, v_{n-2}^j\} \cup \{v_{n-1}, v_n, v_{n-1}^j, v_n^j\}$. Thus $V[S'_m(P_n)]$ can be expressed as a union of $\frac{n}{4}+1$ vertex disjoint subsets.

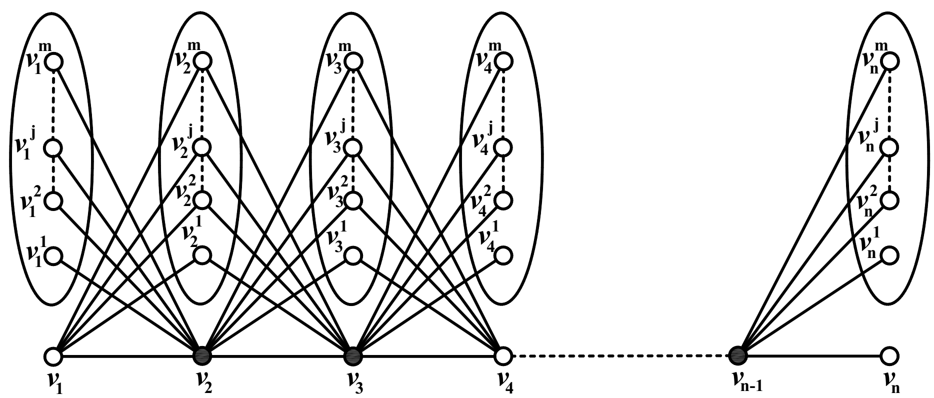
As discussed in Case(i) minimum $\frac{n-2}{2}$ vertices are required to dominate $\frac{n}{4}$ vertex disjoint subsets. Now, we only need to dominate the last vertex disjoint subset $v_{n-1}, v_n, v^j_{n-1}, v^j_n$. The vertices v^j_n are pendant vertices which are adjacent to vertex v_{n-1}. Then it is required to take v_{n-1} in the dominating set. Now, the vertex v_{n-1} dominates v_n, v^j_n in the same vertex disjoint subset as well as v_{n-2} and v^j_{n-2}'s from the previous vertex disjoint subset. The vertices of type v^j_{n-1}'s are not dominated by v_{n-1}. These v^j_{n-1} are adjacent to v_{n-2} of previous vertex disjoint subset and to v_n of the same vertex disjoint subset. So, to dominate these v^j_{n-1} vertices we can either take v_{n-2} or v_n in the dominating set. Let us take v_n in the dominating set. Hence to dominate the last vertex disjoint subset minimum two vertices are required. Hence $|D| \geq \frac{n-2}{2} + 2 = \frac{n}{2} + 1$.

Then the dominating set $D = \{v_2, v_3, v_6, v_7, v_{10}, v_{11} \ldots, v_{n-4}, v_{n-3}, v_{n-1}, v_n\}$. Now, $N[D] = \{v_1, v_2, \ldots, v_n, v^j_1, v^j_2, \ldots, v^j_n\} = V[S'_m(P_n)]$. Hence D is the required dominating set of minimum cardinality. Also, the degree of all the vertices in D is greater than or equal to all the vertices of $V[S'_m(P_n)] - D$. Therefore D is a strong dominating set with minimum cardinality. Thus,

$$\gamma[S'_m(P_n)] = \gamma_{st}[S'_m(P_n)] = \frac{n}{2} + 1, \text{ for } n \equiv 2\,(mod\ 4)$$

Hence

$$\gamma[S'_m(P_n)] = \gamma_{st}[S'_m(P_n)] = \begin{cases} \frac{n}{2} & \text{if } n \equiv 0(mod\ 4) \\ \frac{n+1}{2} & \text{if } n \equiv 1,3(mod\ 4) \\ \frac{n}{2}+1 & \text{if } n \equiv 2(mod\ 4) \end{cases}$$



$S'_m(P_n)$
Figure 2.1

Theorem 2.6. *$S'_m(P_n)$ is $d-$balanced graph.*

Proof: In above Theorem 2.5 we have obtained the strong dominating set D in which every $v \in D$ strongly dominate some $u \in V[S'_m(P_n)] - D$. Then D is $s-full$. Thus by Proposition 2.4, $S'_m(P_n)$ is $d-$balanced.

Corollary 2.7. $\gamma[S'_m(C_n)] = \gamma_{st}[S'_m(C_n)] = \begin{cases} \frac{n}{2} & \text{if } n \equiv 0(mod\ 4) \\ \frac{n+1}{2} & \text{if } n \equiv 1,3(mod\ 4) \\ \frac{n}{2}+1 & \text{if } n \equiv 2(mod\ 4) \end{cases}$

When the terminal vertices of path are identified then the resultant graph is cycle. The proof is similar to Theorem 2.5.

Corollary 2.8. *$S'_m(C_n)$ is d−balanced graph.*

Theorem 2.9. $\gamma[S'_m(K_{p,q})] = \gamma_{st}[S'_m(K_{p,q})] = 2.$

Let $K_{p,q}$ be the complete bipartite graph with $V_1 \cup V_2 = V$. Let the partite set V_1 contain the vertices $v_1, v_2, \ldots, v_q$ and the partite set V_2 contain the vertices $u_1, u_2, \ldots, u_p$. Let v_i^j and u_l^j $(1 \leq j \leq m)$, $(1 \leq i \leq q)$, $(1 \leq l \leq p)$, be the vertices corresponding to each v_i and u_l which are added in the $K_{p,q}$ to obtain $S'_m(K_{p,q})$ such that $N(v_i^1) = N(v_i^2) = \ldots = N(v_i^m)$ and $N(u_l^1) = N(u_l^2) = \ldots = N(u_l^m)$. Then $|V[S'_m(K_{p,q})]| = (m+1)(p+q) = mp + mq + p + q$. Now, $d(v_1) = d(v_2) = d(v_3) = \ldots = d(v_q) = mp + p$ and $d(u_1) = d(u_2) = d(u_3) = \ldots = d(u_p) = mq + q$ while $d(v_i^j) = p$ and $d(u_l^j) = q$. Note that $d(v_i) = mp + p$ and $d(u_i) = mq + q$. So either $d(v_i)$ is maximum or $d(u_i)$ is maximum.

Now $d(v_i) = mp + p$ that is, v_i is adjacent to p vertices of type u_l's and mp vertices of type u_l^j's. Then to dominate these $mp + p$ vertices any one vertex from v_i's is required. Without loss of generality we take v_1 in the dominating set. Thus vertex v_1 dominates $mp+p$ number of vertices. So, we only need to dominate remaining $mp+mq+p+q-mp-p = mq+q$ number of vertices. Now $d(u_i) = mq + q$ that is, u_i is adjacent to q vertices of type v_i's and mq vertices of type v_i^j's. Then to dominate these $mq+q$ vertices any one vertex from u_l's is required. Without loss of generality we take u_1 in the dominating set. Thus vertex u_1 dominates $mq + q$ vertices. Hence $|D| \geq 2$. Then the dominating set $D = \{u_1, v_1\}$. Now $N[D] = \{v_1, v_2 \ldots, v_q, u_1, u_2, \ldots, u_p, v_i^j, u_l^j\} = V[S'_m(K_{p,q})]$. Thus $D = \{v_1, u_1\}$ is the dominating set with minimum cardinality. The dominating set D also forms the strong dominating set with minimum cardinality as both the vertices v_1 and u_1 in D are of maximum degree $mp + p$ and $mq + q$ respectively. Therefore,

$$\gamma[S'_m(K_{p,q})] = \gamma_{st}[S'_m(K_{p,q})] = 2 \quad .$$

Theorem 2.10. *$S'_m(K_{p,q})$ is d− balanced graph.*

In above Theorem 2.9 we have obtained the strong dominating set $D = \{v_1, u_1\}$. The vertices v_1 strongly dominates the vertices of the type u_i's and u_i^j's while u_1 strongly dominates vertices of type v_i's and v_i^j's. Thus D is $s - full$. Thus by Proposition 2.4, $S'_m(K_{p,q})$ is $d-$ balanced.

Theorem 2.11. $\gamma[S'_m(G_n)] = \gamma_{st}[S'_m(G_n)] = \begin{cases} 1 + \dfrac{n}{2} & \text{if } n \text{ is even} \\ 1 + \dfrac{n+1}{2} & \text{if } n \text{ is odd} \end{cases}$

Let $v_1, v_2, \ldots, v_n$ be the vertices of degree three and $u_1, u_2, \ldots, u_n$ be the vertices of degree two while v be the apex. Let v_i^j, u_i^j and v^j be the vertices corresponding to each v_i, u_i and v respectively which are added in the gear graph to obtain $S'_m(G_n)$. Thus $N(v_i^1) = N(v_i^2) = \ldots = N(v_i^m)$, $N(u_i^1) = N(u_i^2) = \ldots = N(u_i^m)$ and $N(v^1) = N(v^2) = \ldots = N(v^m)$. Hence $|V(G_n)| = (m+1)(2n+1) = 2nm + m + 2n + 1$.

The degree $d(v_i) = 3(m+1)$, $d(u_i) = 2(m+1)$, $d(v) = n(m+1) = \triangle(G_n)$, $d(v_i^j) = 3$, $d(u_i^j) = 2$ and $d(v^j) = n$. The vertex v must be in any dominating set as $d(v) = n(m+1) = \triangle(G_n)$. The vertex v dominates n vertices of type v_i's and nm vertices of type v_i^j's alongwith itself. There are remaining $2nm + m + 2n + 1 - n - nm - 1 = nm + n + m$. Now to dominate these remaining vertices we consider following cases.

Case(i) : n is even
The nm vertices of type u_i^j's are of degree two. That is they are adjacent to v_i and v_{i-1} vertices only. In this case, each v_i can dominate $2m$ vertices of type u_i^j's and u_{i+1}^j's. So, to dominate nm vertices of type u_i^j's we require minimum $\frac{nm}{2m} = \frac{n}{2}$ number of vertices of type v_i's. These $\frac{n}{2}$ vertices of type v_i's will dominate n vertices of type u_i's, nm vertices of type u_i^j's and m vertices of type v^j's. Thus the dominating set contains $\frac{n}{2}$ vertices of type v_i's and apex v. If D is the dominating set then $|D| \geq \frac{n}{2} + 1$. Hence $N[D] = \{v, v_i, u_i, v^j, v_i^j, u_i^j\} = V(G_n)$. Therefore D forms the dominating set of minimum cardinality. The degree of all the vertices in D is greater than or equal to the degree of all the vertices in $V(G_n) - D$. Hence D also forms the strong dominating set of minimum cardinality. Thus,

$$\gamma[S'_m(G_n)] = \gamma_{st}[S'_m(G_n)] = 1 + \frac{n}{2}; \text{ for } n \text{ even.}$$

Case(ii) : n is odd
The nm vertices of type u_i^j's are of degree two. That is they are adjacent to v_i and v_{i-1} vertices only. In this case, there are $n-1$ even number of vertices. Now each v_i can dominate $2m$ vertices of type u_i^j's and u_{i+1}^j's. So, to dominate $(n-1)m$ even number of vertices of type u_i^j's we require minimum $\frac{m(n-1)}{2m} = \frac{n-1}{2}$ number of vertices of type v_i's as discussed in Case(i).

These $\frac{n-1}{2}$ vertices of type v_i's will dominate $(n-1)$ vertices of type u_i's, $(n-1)m$ vertices of type u_{n-1}^j's and m vertices of type v^j's. Now we only need to dominate m vertices of type u_n^j's and vertex u_n. Therefore it is required to take either v_n or v_{n-1} in any dominating set. Thus the dominating set contains $\frac{n+1}{2}$ vertices of type v_i's and apex v.

If D is the dominating set then $|D| \geq \frac{n+1}{2} + 1$. Then $N[D] = \{v, v_i, u_i, v^j, v_i^j, u_i^j\} = V(G_n)$. Therefore D forms the dominating set of minimum cardinality. The degree of all the vertices in D is greater than or equal to the degree of all the vertices in $V(G_n) - D$. Hence D also forms the strong dominating set of minimum cardinality. Thus,

$$\gamma[S'_m(G_n)] = \gamma_{st}[S'_m(G_n)] = 1 + \frac{n+1}{2}; \text{ for } n \text{ odd}$$

Theorem 2.12. *$S'_m(G_n)$ is $d-$balanced graph.*

In above Theorem 2.11 we have obtained the strong dominating set D. The vertices $v \in D$ strongly dominate some vertex $u \in V[S'_m(P_n)] - D$. Thus D is $s-full$. Thus by Proposition 2.4 $S'_m(P_n)$ is $d-$balanced.

Observation 2.13. $\gamma[S'_m(K_n)] = \gamma_{st}[S'_m(K_n)] = 2$.

Observation 2.14. *$S'_m(K_n)$ is $d-$balanced graph.*

Observation 2.15. $\gamma[S'_m(K_{1,n})] = \gamma_{st}[S'_m(K_{1,n})] = 2$.

Observation 2.16. *$S'_m(K_{1,n})$ is $d-$balanced graph.*

3 Concluding Remarks

The concept of splitting in graphs is useful to construct larger graph from a given graph. The concept of m- splitting is more generalised way to contruct larger graph from the given graph because all the vertices are duplicated m times. We have obtained several results on strong domination in graphs in the context of m - splitting of graphs.

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