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Spherically Symmetric Cosmological Model with Charged Anisotropic Fluid in Rosen's Bimetric Theory of Gravitation

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Abstract

In this paper we have obtained cosmological models for the static spherically symmetric space-time with charged anisotropic fluid distribution in context of Rosen's Bimetric General Relativity (BGR). An exact solution is obtained and a special case is considered.

Keywords: Bimetric Relativity, Charged anisotropic fluid, Exact solution of field equations.
AMS Classification: 83D05

1 Introduction

The understanding of gravitation is a problem of concern to many researchers. Many theories are available for the explanation. Among the successful theories Newton's theory (in limited sense) and Einstein's theory are leading. However, new theories either due to modification or with new ideas are continuously proposed. To avoid the singularities occurring into the solutions of field equations, an attempt is made by Rosen 1940, 1963, 1973 [1] [2] [3] [4] who proposed a modification of the general relativity known as Bimetric theory of gravitation. This theory has attracted good number of researchers to study its various aspects and several research papers were published on this theory in subsequent years. In last decade Khadekar and Tade 2007 [13], Sahoo 2008 [14], Tripathy, Behera and Sahoo 2010 [15], Mahurpawar and Ronghe 2011 [16], Sahoo, Mishra and Ramu 2011 [17], Jain, Sahoo and Mishra 2012 [18], Sahoo and Mishra 2013, 2014 [19] [20] [21] [22] [23], Sahu, Misra and Behera 2015 [24], Borkar and Gayakwad 2017 [25] etc. have investigated various cosmological models based on different space-times in the context of this Bimetric theory of gravitation. However, in most of the cases only vacuum solutions exist and so we can conclude that this bimetric theory of gravitation does not help in any way to describe the early era of the universe.

Hence to explore the possibilities of exact solutions of field equations here we have considered a modified version of the previous bimetric theory, called the Bimetric General Relativity (BGR), proposed by Rosen 1978, 1980 [5] [6] [7]. In this theory gravity is attributed to a curved space-time described by the metric,

$$ds^2 = g_{ij}dx^i dx^j, \tag{1}$$

a second metric tensor in the background space is described by

$$d\sigma^2 = \gamma_{ij}dx^i dx^j. \tag{2}$$

The field equations in BGR are written in the form of Einstein's field equations, but with an additional term on the right hand side,

$$G^{\mu}_{\nu} = -8\pi T^{\mu}_{\nu} + S^{\mu}_{\nu}, \tag{3}$$

where G^{μ}_{ν} is the Einstein tensor, T^{μ}_{ν} energy-momentum tensor of matter distribution and

$$S^{\mu}_{\nu} = \frac{3}{a^2}(\gamma_{\nu\alpha}g^{\alpha\mu} - \frac{1}{2}\delta^{\mu}_{\nu}g^{\alpha\beta}\gamma_{\alpha\beta}), \tag{4}$$

where a is a constant scale parameter. The order of this scale parameter is related to the size of the universe.

In this paper we present an exact solution of the field equations for charged anisotropic fluid in context of BGR proposed by Rosen 1980 [6]. We have followed the method developed by Khadekar & Kandalkar 2004 [11] by introducing the *generating function* $G(r)$ which determines the relevant physical variables as well as the metrical coefficient and a function $w(r)$ measuring the degree of anisotropy, this function is called *anisotropic function.* The General Relativity analogue for the charged anisotropic fluid was considered by Singh *et al.* 1995 [10] and results obtained here match with those of obtained there. Also in absence of charge, results obtained in this paper match with the one obtained by Khadekar & Kandalkar 2004 [11] for Rosen's Bimetric Theory of Gravitation.

1.1 Metrics and Field Equations

The general static spherically symmetric line element may be expressed as

$$ds^2 = -e^{\lambda(r)}dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2) + e^{\nu(r)}dt^2. \tag{5}$$

Consider the background flat metric $\gamma_{\mu\nu}$ in a static de-Sitter form as

$$d\sigma^2 = -\left(1 - \frac{r^2}{a^2}\right)^{-1}dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2) + \left(1 - \frac{r^2}{a^2}\right)dt^2. \tag{6}$$

For a region very small compared to a, i.e. for $r \ll a$, this line element has Minkowski form

$$d\sigma^2 = -dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2) + dt^2. \tag{7}$$

The convention used here for coordinates is

$$x^1 = r, x^2 = \theta, x^3 = \phi, x^4 = t.$$

The energy momentum tensor for charged anisotropic fluid is of the form

$$T_{\mu\nu} = (\rho + p)U_{\mu}U_{\nu} - pg_{\mu\nu} + \pi_{\mu\nu} + \frac{1}{4\pi}\{-F^{\mu\lambda}F^{\nu}_{\lambda} + \frac{1}{4}g^{\mu\nu}(F^{\lambda\alpha}F_{\lambda\alpha})\}, \tag{8}$$

with matter density ρ, isotropic pressure p, U_{μ} being the four-velocity vector of the fluid, and the anisotropic stress tensor $\pi_{\mu\nu}$ is given by

$$\pi_{\mu\nu} = \sqrt{3}S[c_{\mu}c_{\nu} - \frac{1}{3}(U_{\mu}U_{\nu} - g_{\mu\nu})], \tag{9}$$

where $S = S(r)$ is the magnitude of the anisotropic stress tensor and the unit space-like radial vector c^{μ} is obtained as

$$c^{\mu} = (-e^{-\frac{\lambda}{2}}, 0, 0, 0), \tag{10}$$

so that $c^{\mu}c_{\mu} = -1$. The skew symmetric Maxwell Tensor $F_{\mu\nu}$ satisfies the Maxwell's equations in the form

$$F_{\mu\nu;\lambda} + F_{\nu\lambda;\mu} + F_{\lambda\mu;\nu} = 0, \tag{11}$$

$$F^{\mu\nu}{}_{;\nu} = -4\pi J^{\mu}, \tag{12}$$

where $J^{\mu} = \sigma U^{\mu}$ is the 4-current of the charge distribution with proper charge density σ within the sphere. It is known that due to spherical symmetry the only non-vanishing component of $F_{\mu\nu}$ is,

$$F_{14} = -\exp\{(\lambda+\nu)/2\}Q(r)/r^2, \tag{13}$$

where

$$Q(r) = 4\pi \int_0^r x^2 e^{\lambda/2}\sigma(x)dx, \tag{14}$$

represents the total charge contained within a sphere of coordinate radius r.
Choosing the comoving system we write

$$U^{\mu} = (0,0,0,e^{-\frac{\nu}{2}}), \tag{15}$$

here $U_{\mu}U^{\mu} = 1$.
The non-vanishing components of the energy momentum tensor are

$$T_1^1 = -\left(p + \frac{2S}{\sqrt{3}}\right) + \frac{Q^2}{8\pi r^4}, \tag{16}$$

$$T_2^2 = T_3^3 = -\left(p - \frac{S}{\sqrt{3}}\right) - \frac{Q^2}{8\pi r^4}, \tag{17}$$

$$T_4^4 = \rho + \frac{Q^2}{8\pi r^4}. \tag{18}$$

The pressure along radial direction and tangential directions are,

$$p_r = \left(p + \frac{2S}{\sqrt{3}}\right), \tag{19}$$

and

$$p_{\perp} = \left(p - \frac{S}{\sqrt{3}}\right), \tag{20}$$

respectively.
From equations (19) and (20),the magnitude of anisotropic stress tensor is

$$S = \left(\frac{p_r - p_{\perp}}{\sqrt{3}}\right). \tag{21}$$

In the region $r \ll a$ and neglecting the terms which are small throughout this region, we can write the non-vanishing components of S_{μ}^{ν} (Falik & Rosen 1980 [8]) as,

$$-S_1^1 = -S_2^2 = -S_3^3 = S_4^4 = \frac{3}{2a^2}e^{-\nu}. \tag{22}$$

The computation of Einstein tensor G_{ν}^{μ} for metric (5) was done using Mathematica program described in Hasmani & Rathva 2007 [12] with appropriate modification. The other part of the field equations (3) is computed using the energy-momentum tensor (8), background metric (7) and values from equation (22). The procedure adopted here is as given in Rosen 1980 [6]. Thus the final form of field equations are written as,

$$e^{-\lambda}\left[\frac{\nu'}{r} + \frac{1}{r^2}\right] - \frac{1}{r^2} = 8\pi p_r - \frac{Q^2}{r^4} - \frac{3}{2a^2}e^{-\nu}, \tag{23}$$

$$e^{-\lambda}\left[\frac{\nu''}{2} - \frac{\lambda'\nu'}{4} + \frac{\nu'^2}{4} - \frac{(\lambda'-\nu')}{2r}\right] = 8\pi p_{\perp} + \frac{Q^2}{r^4} - \frac{3}{2a^2}e^{-\nu}, \tag{24}$$

$$e^{-\lambda}\left[\frac{-\lambda'}{r} + \frac{1}{r^2}\right] - \frac{1}{r^2} = -8\pi\rho - \frac{Q^2}{r^4} + \frac{3}{2a^2}e^{-\nu}. \tag{25}$$

Here a prime for λ and ν denotes a differentiation with respect to r.
The energy-momentum conservation equation $T^{\mu}_{\nu;\mu} = 0$ gives,

$$(\rho + p_r)\nu' + 2p_r' + 4\sqrt{3}\frac{S}{r} = \frac{1}{4\pi r^4}\frac{dQ^2}{dr}. \tag{26}$$

We now define the *effective* density ρ_e, *effective* radial pressure p_{r_e} and *effective* tangential pressure $p_{\perp_e}$ (Harpaz & Rosen 1985 [9]) as,

$$\begin{aligned} \rho_e =& \quad \rho - \frac{3}{16\pi a^2}e^{-\nu}, \\ p_{r_e} =& \quad p_r - \frac{3}{16\pi a^2}e^{-\nu}, \\ p_{\perp_e} =& \quad p_\perp - \frac{3}{16\pi a^2}e^{-\nu}. \end{aligned} \tag{27}$$

So the field equations (23)-(25) take the form,

$$e^{-\lambda}\left[\frac{\nu'}{r} + \frac{1}{r^2}\right] - \frac{1}{r^2} = 8\pi p_{r_e} - \frac{Q^2}{r^4}, \tag{28}$$

$$e^{-\lambda}\left[\frac{\nu''}{2} - \frac{\lambda'\nu'}{4} + \frac{\nu'^2}{4} - \frac{(\lambda' - \nu')}{2r}\right] = 8\pi p_{\perp_e} + \frac{Q^2}{r^4}, \tag{29}$$

$$e^{-\lambda}\left[\frac{\lambda'}{r} - \frac{1}{r^2}\right] + \frac{1}{r^2} = 8\pi\rho_e + \frac{Q^2}{r^4}. \tag{30}$$

Equation (26) can be rewritten as,

$$(\rho_e + p_{r_e})\nu' + 2p_{r_e}' + 4\sqrt{3}\frac{S}{r} = \frac{1}{4\pi r^4}\frac{dQ^2}{dr}. \tag{31}$$

Now from equation (30),

$$e^{-\lambda} = 1 - \frac{2m_e(r)}{r} + \frac{Q^2}{r^2}, \tag{32}$$

where $m_e(r)$ is the *effective* mass function defined as,

$$m_e(r) = \int_0^r (4\pi\rho_e r^2 + \frac{QQ'}{r})dr. \tag{33}$$

Now from equation (31),

$$\nu' = -\frac{2p_{r_e}'}{(\rho_e + p_{r_e})} - \frac{4\sqrt{3}S}{r(\rho_e + p_{r_e})} + \frac{1}{4\pi r^4(\rho_e + p_{r_e})}\frac{dQ^2}{dr}. \tag{34}$$

Using equations (32) and (34) in (28), one can get

$$\begin{aligned} &\left(1 - \frac{2m_e}{r} + \frac{Q^2}{r^2}\right)\cdot\left(-\frac{2rp_{r_e}'}{(\rho_e + p_{r_e})} - \frac{4\sqrt{3}S}{(\rho_e + p_{r_e})} + \frac{1}{4\pi r^3(\rho_e + p_{r_e})}\frac{dQ^2}{dr} + 1\right) \\ &= 8\pi p_{r_e}r^2 + 1 - \frac{Q^2}{r^2}. \end{aligned} \tag{35}$$

Define a *generating function* $G(r)$ as,

$$G(r) = \frac{1 - \frac{2m_e}{r} + \frac{Q^2}{r^2}}{8\pi p_{r_e}r^2 + 1 - \frac{Q^2}{r^2}}, \tag{36}$$

and introduce the *anisotropic function* $w(r)$ as,

$$w(r) = \frac{4\sqrt{3}S}{(\rho_e + p_{r_e})}G(r). \tag{37}$$

Using equations (36) and (37) in equation (35), we get

$$8\pi(\rho_e + p_{r_e}) = \frac{-16\pi r p'_{r_e} G + \frac{4QQ'G}{r^3}}{(1 - G + w)}. \tag{38}$$

Differentiation of equation (32) gives,

$$e^{-\lambda}\lambda' = \frac{2m'_e}{r} - \frac{2m_e}{r^2} - \frac{2QQ'}{r^2} + \frac{2Q^2}{r^3}. \tag{39}$$

Adding $8\pi p_{r_e}$ on both sides of equation (30) and then using equations (39), (32) and (36), we get

$$8\pi(\rho_e + p_{r_e}) = \frac{2m'_e}{r^2} - \frac{2m_e}{r^3} - \frac{2QQ'}{r^3} + \frac{2Q^2}{r^4} + \left(\frac{1}{r^2} + 8\pi p_{r_e} - \frac{Q^2}{r^4}\right)(1 - G). \tag{40}$$

Differentiation of equation (36) gives,

$$\begin{aligned}\frac{2m'_e}{r^2} - \frac{2m_e}{r^3} - \frac{2QQ'}{r^3} + \frac{2Q^2}{r^4} = &- \frac{G'(r)}{r}\left(8\pi p_{r_e} r^2 + 1 - \frac{Q^2}{r^2}\right) - 8\pi p'_{r_e} r G \\ &- 16\pi p_{r_e} G + \frac{2QQ'G}{r^3} - \frac{2Q^2 G}{r^4}.\end{aligned} \tag{41}$$

Using equations (41) and (38) into equation (40), we get

$$\begin{aligned}8\pi p'_{r_e} + \frac{(1 - 3G - rG')(1 - G + w)}{rG(1 + G - w)} 8\pi p_{r_e} + \frac{(1 - G - rG')(1 - G + w)}{r^3 G(1 + G - w)} \\ - \frac{(1 + G - rG')(1 - G + w)}{rG(1 + G - w)}\frac{Q^2}{r^4} - \frac{2QQ'}{r^4} = 0,\end{aligned} \tag{42}$$

which is a linear equation in p_{r_e}. We obtain its solution as,

$$8\pi p_{r_e} = e^{-\int B(r)dr}[p_0 + \int C(r) e^{\int B(r)dr} dr], \tag{43}$$

where p_0 is a constant of integration,

$$B(r) = \frac{(1 - 3G - rG')(1 - G + w)}{rG(1 + G - w)},$$

$$C(r) = -\frac{(1 - G - rG')(1 - G + w)}{r^3 G(1 + G - w)} + \frac{(1 + G - rG')(1 - G + w)Q^2}{r^5 G(1 + G - w)} + \frac{2QQ'}{r^4}.$$

From equation (33),

$$\frac{m'_e}{r} = 4\pi\rho_e r + \frac{QQ'}{r^2}. \tag{44}$$

Using equation (44) in (41) and then using equation (36), we get

$$8\pi\rho_e = (1 - G)\frac{1}{r^2} - (1 + G)\frac{Q^2}{r^4} - 8\pi G\left(3p_{r_e} + r p'_{r_e} - \frac{QQ'}{4\pi r^3}\right) - rG'\left(8\pi p_{r_e} + \frac{1}{r^2} - \frac{Q^2}{r^4}\right), \tag{45}$$

which is the expression for effective density ρ_e. Equation (37) gives,

$$S = \frac{w(r)}{4\sqrt{3}G(r)}(\rho_e + p_{r_e}). \tag{46}$$

Equations (32) and (36) yields,

$$e^{-\lambda} = G(r)(8\pi p_{r_e} r^2 + 1 - \frac{Q^2}{r^2}). \tag{47}$$

Using this above equation in equation (28), we obtain

$$\frac{d\nu}{dr} = \frac{1}{rG(r)} - \frac{1}{r}, \tag{48}$$

which after integration gives,

$$e^{\nu} = \frac{A^2}{r} e^{\int (1/rG)dr}, \tag{49}$$

where A is the constant of integration.
Using values from equations (49) and (32), the space-time (5) becomes,

$$ds^2 = -\left[1 - \frac{2m_e}{r} + \frac{Q^2}{r^2}\right]^{-1} dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2) + \frac{A^2}{r} e^{\int (\frac{1}{rG})dr} dt^2. \tag{50}$$

2 Special case:

To make the solution physically significant, the generating function G must satisfy some conditions. Thus, assuming the non divergent effective pressure at origin, as $r \to 0$, $\frac{m_e(r)}{r} \to 0$ and $\frac{Q^2}{r^2} \to 0$ which imply that $G(r) \to 1$. If $G(r) = 1, w(r) = 0 (i.e. p_r = p_\perp)$ and $Q(r) = 0$, one obtains Minkowski flat space-time.
By considering the charge density σ to be constant, we can get $Q(r) \sim r^3$ from equation (14). The appropriate junction condition at the surface $r = r_0$ yields

$$Q(r) = e(r/r_0)^3. \tag{51}$$

If we denote $e/r_0^3 = K$ then we can write,

$$Q(r) = kr^3. \tag{52}$$

Further we define *Generating function* and *Anisotropic function* from equations (36) and (37) respectively as,

$$G(r) = 1 - \alpha r^2, \tag{53}$$

$$w(r) = -\alpha r^2, \tag{54}$$

where α is a constant such that $G - w = 1 \neq 0$. This choice is physically appropriate because function $G(r) \sim 1$ as $r \sim 0$.
From equation (44), $B(r) = 0, C(r) = 6K^2 r$. So from equation (43),

$$p_{r_e} = \frac{p_0}{8\pi} + \frac{3K^2 r^2}{8\pi}. \tag{55}$$

If constant $p_0 = 0$, then

$$p_{r_e} = \frac{3K^2 r^2}{8\pi}. \tag{56}$$

Hence from equation (27) the radial pressure is given by,

$$p_r = \frac{3}{16\pi a^2}\left(\frac{\sqrt{1-\alpha r^2}}{A^2}\right) + \frac{3K^2 r^2}{8\pi}. \tag{57}$$

Also from equation (45) the effective density is obtained as,

$$\rho_e = \frac{3\alpha}{8\pi} - \frac{K^2 r^2}{8\pi}(11 - 14\alpha r^2), \tag{58}$$

which gives the energy density using equation (27) as,

$$\rho = \frac{3}{16\pi a^2}\left(\frac{\sqrt{1-\alpha r^2}}{A^2}\right) + \frac{3\alpha}{8\pi} - \frac{K^2 r^2}{8\pi}(11 - 14\alpha r^2). \tag{59}$$

Using values from equations (53), (54), (56) and (58) into equation (46), one can obtain

$$S = \frac{1}{8\pi\sqrt{3}}\left[\frac{-3\alpha^2 r^2 - 14\alpha^2 K^2 r^6 + 8\alpha K^2 r^4}{4(1-\alpha r^2)}\right]. \tag{60}$$

Using equations (53) and (56), equation (47) can be written as

$$e^{-\lambda} = (1 - \alpha r^2)(1 + 2K^2 r^4). \tag{61}$$

Using equation (53) into equation (49), we get

$$e^{\nu} = \frac{A^2}{\sqrt{1 - \alpha r^2}}. \tag{62}$$

Using values from equations (61) and (62), the cosmological model for space-time (5) is

$$ds^2 = -[(1 - \alpha r^2)(1 + 2K^2 r^4)]^{-1} dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2) + \frac{A^2}{\sqrt{1 - \alpha r^2}} dt^2. \tag{63}$$

3 Conclusion

In this paper we have presented exact analytical solution of field equations of Bimetric General Relativity for the case of static spherically symmetric anisotropic distribution of charged matter by introducing the *generating function* and *anisotropic function* as defined in (53) and (54) so that $G - w = 1 \neq 0$. From equation (33), we note that besides material density the electromagnetic anisotropy also contributes to the mass. It can also be noted that for $Q(r) = 0$, the solution obtained here match with the solution of Khadekar & Kandalkar 2004 [11] for a neutral matter; thus the results obtained here are more general. Moreover the present result reduces to the Einstein's general relativity for a physical system which is small compare to the size of the universe because in such case the term $\frac{3}{2a^2}e^{-\nu}$ in the field equations (23)-(25) is negligible, this conclusion is derived by matching our results with the one obtained by Singh *et al.* (1995) [10] for the General Relativity.

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