Two-level trade credit policy for retailer’s optimal ordering policy for
deteriorated items in fuzzy system

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(Acceptance Date: Jan 6, 2015)

Abstract

In this article, a fuzzy mathematical model for deteriorating items is developed to formulate optimal ordering policies for the retailer under the conditions of permissible delay in payments, in which the supplier offers a permissible delay period to the retailer and the retailer in turn provides a trade credit period to their customers. The demand rate, ordering cost, selling price per item and deterioration rate are taken as fuzzy numbers. The total variable cost in fuzzy sense is de-fuzzified using the centre of gravity method. A numerical example demonstrates the applicability of the proposed model.

Key words: Inventory, Two-level credit policy, Deterioration, Delay in payments, Centre of gravity method, Fuzzy set theory.

1. Introduction and related literature

In real life business, we observe that the trade credit financing becomes a powerful tool to improve sales and profits in an industry. A profitable decision policy between a supplier and the retailers can be characterized by an agreement on the trade credit scenario such as permissible delay in payments. In practice, a supplier frequently offers a retailer a delay of a fixed time period (say, 30 days) for settling the amount owed to him. Usually, there is no interest charge if the outstanding amount is paid with in the permissible delay period. However, if the payment is not paid in full by the end of the permissible delay period, then interest is charged on the outstanding amount. Therefore, it is clear that a customer will delay the payment up to the last moment of the permissible period allowed by the supplier. The
permissible delay in payments produces two benefits to the supplier: first is, it attracts new customers who consider it to be a type of price reduction, and second is, it may be applied as an alternative to price discount because it does not provoke competitors to reduce their prices and thus introduce lasting price reduction. On the other hand, the policy of granting credit terms not only an additional cost but also an additional dimension of default risk to the supplier.

Inventory models with deteriorating items have been studied by researchers from time to time. Deterioration of an item occurs in the cases such as decay, evaporation, obsolescence, loss of utility or marginal value of a commodity, gasoline, fertilizers, different type of oils, milks, medicines etc.

Goyal (1985) first studied an EOQ model under the conditions of permissible delay in payments. He assumes that the supplier would offer the retailer a fixed delay period and the retailer could sell the goods and accumulate revenue and earn interest with in the trade credit period. He implicitly assumes that the customer would pay for the items as soon as the items are received from the retailer. That is, he assumed that the supplier would offer the retailer a delay period but the retailer would not offer the trade credit period to customers. Shah (1993a, 1993b), Aggarwal and Jaggi (1995), Hwang and Shinn (1997) extended Goyal’s (1985) model to consider the deterministic inventory model with a constant deterioration rate. Shinn et al. (1996) extended Goyal’s (1985) model and considered quantity discount for freight cost. Jamal et al. (1997) extended Aggarwal and Jaggi (1995) model to allow for shortages.

Chung (1998) presented the DCF (discounted cash flow) approach for the analysis of the optimal inventory policy in the presence of trade credit. Shah and Shah (1998) developed a probabilistic inventory model when delay in payment is permissible. Jamal et al. (2000) and Sarker at al. (2000) computed interest earned on the selling price and concluded that the retailer should settle his account relatively sooner as the unit-selling price increases relative to the unit purchase cost. Chang and Dye (2001) extended the model of Jamal et al. (1997) for time dependent deterioration. They assumed that the backlogging rate is inversely proportional to the waiting time.

Teng (2002) proved that it is beneficial for a well-established retailer to put order of smaller size and take the benefits of the permissible delay more frequently. Shinn and Hwang (2003) determined the retailer’s optimal price and order size simultaneously under the condition of order-size-dependent delay in payment. Huang and Chung (2003) extended Goyal’s (1985) model to discuss the replenishment and payment policies to minimize the
annual total average cost under cash discount and payment delay from the retailer’s point of view. Chang et al. (2003) determined an economic order quantity model for deteriorating items in which the supplier offer a trade credit to the retailer if the order quantity is greater than or equal to a pre-specified quantity. Teng et al. (2005) developed the optimal pricing and lot sizing under permissible delay in payments by considering the difference between selling price and purchase quantity and demand to be price sensitive.

All the above mentioned inventory models implicitly assumed one-level trade credit financing. But, in most business transactions, this assumption is unrealistic and usually the supplier offers a credit period to the retailer and the retailer, in turn, passes on this credit period to his/her customers. Recently, researchers developed inventory models under the two-level trade credit financing.

Huang (2003) presented an inventory model assuming that the retailer also offers a credit period to his/her customer which is shorter than the credit period offered by the supplier, in order to stimulate the demand. Huang (2006) extended Huang’s (2003) model to investigate the retailer’s inventory policy under two levels of trade credit and limited storage space. Huang (2007) incorporated Huang (2003) to investigate the two-level trade credit policy in the EPQ framework. Huang and Hsu (2008) have developed an inventory model under two-level trade credit policy by incorporating partial trade credit option at the customers of the retailer.

Usually researchers consider different parameters of an inventory model either as constant or dependent on time or probabilistic in nature for the development of the economic order quantity model. But, in the real life situations, these parameters may have little deviations from the exact value, which may not follow any probability distribution. In these situations, if these parameters are treated as fuzzy parameters, then it will be more realistic. These types of problems are de-fuzzified first using a suitable fuzzy technique and then the solution procedure can be obtained in the usual manner. Several authors, namely Chang et al. (1998), Lee and Yao (1998), Lin and Yao (2000), Yao et al. (2000), De, Kundu and Goswami (2003), De and Goswami (2006) and Gani and Maheswari (2010) developed inventory models in fuzzy sense by considering different parameters as fuzzy parameters.

In this paper, we propose an economic order quantity model for deteriorating items under two-level trade credit policy in the fuzzy sense. The demand rate, ordering cost; selling price per item and deterioration rate may be flexible with some vagueness for their values. In real life situations, all these parameters in an inventory model are uncertain, imprecise and the
determination of optimum cycle time becomes a non-stochastic vague decision-making process. In this situation, a suitable way to model these imprecise data to use fuzzy sets, and then the model is formulated in a fuzzy environment. We use the centre of gravity method for de-fuzzifying fuzzy total average cost. Numerical examples are used to illustrate the results given in this paper.

2. Assumptions and Notations

The proposed mathematical model is based on the following assumptions:
(a) The inventory system under consideration deals with a single item.
(b) Replenishment rate is infinite.
(c) Shortages are not allowed.
(d) The lead-time is zero or negligible.
(e) Unit selling price is greater than unit purchasing price.
(f) The deteriorated units can neither be replaced nor repaired during the cycle time.
(g) $I_c$, the interest charge per $ in stocks per year by the supplier; $I_e$, the interest earned per $ per year where $I_c \geq I_e$.
(h) $M$, the retailer’s trade credit period offered by the supplier in years and $N$, the customer’s trade credit period offered by the retailer in years. It is assumed that $M \geq N$.
(i) When $T \geq M$, the account is settled at time $T = M$ and retailer starts paying for the interest charges on the items in stock with rate $I_c$. When $T \leq M$, the account is settled at $T = M$ and the retailer does not need to pay interest charges.
(j) The retailer can accumulate revenue and earn interest after his/her customer pays for the amount of purchasing cost to the retailer until the end of the trade credit period offered by the supplier. That is, the retailer can accumulate revenue and earn interest during the period $N$ to $M$ with rate $I_c$ under the condition of trade credit.

In addition, the following notations are used throughout this paper:

$R$ annual constant demand
$\tilde{R}$ fuzzy annual demand
$A$ ordering cost per order
$\tilde{A}$ fuzzy ordering cost
$P$ selling price per unit
$\tilde{P}$ fuzzy unit selling price
$\theta$ deterioration rate per unit time
\( \tilde{\theta} \)  
Fuzzy deterioration rate per unit time

\( C \)  
Unit purchase cost

\( h \)  
Unit inventory holding cost per year excluding the interest charges

\( Q \)  
Order quantity

\( M \)  
The offered trade credit by the supplier to the retailer to settle the account

\( N \)  
The trade credit period for customers offered by the retailer

\( I_c \)  
Interest charged per $ in stock per year by the supplier

\( I_e \)  
Interest earned per $ per year by the retailer

\( T \)  
The cycle time in years

\( q(t) \)  
The inventory level at any instant of time \( t, 0 \leq t \leq T \)

\( K_1(T) \)  
The total relevant cost per unit time when \( M \leq T \)

\( K_2(T) \)  
The total relevant cost per unit time when \( N \leq T \leq M \)

\( K_3(T) \)  
The total relevant cost per unit time when \( N \geq T \)

\( \tilde{K}_1(T) \)  
Fuzzy total relevant cost per unit time when \( M \leq T \)

\( \tilde{K}_2(T) \)  
Fuzzy total relevant cost per unit time when \( N \leq T \leq M \)

\( \tilde{K}_3(T) \)  
Fuzzy total relevant cost per unit time when \( N \geq T \)

\( U(\tilde{K}_1(T)) \)  
De-fuzzified value of fuzzy total cost \( \tilde{K}_1(T) \)

\( U(\tilde{K}_2(T)) \)  
De-fuzzified value of fuzzy total cost \( \tilde{K}_2(T) \)

\( U(\tilde{K}_3(T)) \)  
De-fuzzified value of fuzzy total cost \( \tilde{K}_3(T) \)

\( T_1 \)  
The total cycle time when \( M \leq T \)

\( T_2 \)  
The total cycle time when \( N \leq T \leq M \)

\( T_3 \)  
The total cycle time when \( N \geq T \)

\( \tilde{T}_1 \)  
Fuzzy total cycle time when \( M \leq T \)

\( \tilde{T}_2 \)  
Fuzzy total cycle time when \( N \leq T \leq M \)

\( \tilde{T}_3 \)  
Fuzzy total cycle time when \( N \geq T \)

\( U(\tilde{T}_1) \)  
De-fuzzified value of fuzzy total cycle time \( \tilde{T}_1 \)

\( U(\tilde{T}_2) \)  
De-fuzzified value of fuzzy total cycle time \( \tilde{T}_2 \)

\( U(\tilde{T}_3) \)  
De-fuzzified value of fuzzy total cycle time \( \tilde{T}_3 \)
3. Crisp mathematical model

Let \( q(t) \) be the inventory level at any time \( t (0 \leq t \leq T) \). Initially, the stock level is \( Q \). The inventory level decreases due to demand and deterioration until it becomes zero at time \( t = T \). The rate of change of inventory is governed by the following differential equation:

\[
\frac{dq(t)}{dt} + \theta q(t) = -R, \quad 0 \leq t \leq T
\]  

(1)

With initial and boundary conditions

\[ q(0) = Q \text{ and } q(T) = 0 \]  

(2)

The solution of (1) is,

\[ q(t) = \frac{R}{\theta} \left( e^{\theta(T-t)} - 1 \right), \quad 0 \leq t \leq T \]  

(3)

Using the eq. (2), the order quantity can be obtained as

\[ Q = q(0) = \frac{R}{\theta} \left( e^{\theta T} - 1 \right) \]  

(4)

Now, the cost of placing an order,

\[ OC = \frac{A}{T} \]  

(5)

The inventory holding cost is given as,

\[ IHC = \frac{hR}{\theta^2 T} \left( e^{\theta T} - \theta T - 1 \right) \]  

(6)

The cost due to deterioration of items is given as,

\[ CD = C(Q - RT) \]

\[ = \frac{CR}{\theta T} \left( e^{\theta T} - \theta T - 1 \right) \]  

(7)

Regarding interest charged and interest earned, based on the length of the cycle time \( T \), three cases may arise:

Case-1: \( M \leq T \)

Case-2: \( N \leq T \leq M \)
Case-3: \( N \geq T \)

These three cases are graphed in figure-1, 2 and 3 respectively.

Case-1: \( M \leq T \)

During the credit period, the retailer sells items and deposits the generated revenue into an amount bearing account at the interest rate \( I_e \) per dollar per year.

Therefore, the interest earned per unit time is,

\[
IE_1 = \frac{PL_e}{T} \int_N^M Rtdt = \frac{PL_e R(M^2 - N^2)}{2T} \quad (8)
\]

The unsold items in stock are charged at interest rate \( I_i \) by the supplier at the beginning of time \( T \). Therefore, the interest charged per unit time is,

\[
IC_1 = \frac{CI_i}{T} \int_M^T q(t)dt = \frac{CI_i}{T} \int_M^T \left( e^{R \theta (T-t)} - 1 \right) dt = \frac{CI_i R}{\theta^2 T} \left[ e^{\theta(T-M)} - \theta(T-M) - 1 \right] \quad (9)
\]

Hence, the total cost per time unit is,

\[
K_1(T) = OC + IHC + CD + IC_1 - IE_1 \quad (10)
\]

Case-2: \( N \leq T \leq M \)

In this case, the total interest earned per unit time is,

\[
IE_2 = \frac{PL_e}{T} \left[ \int_N^T Rtdt + RT(M - T) \right] = \frac{PL_e R}{2T} \left( 2MT - N^2 - T^2 \right) \quad (11)
\]

In this case, total interest charges = 0

Hence, the total cost per time unit is,

\[
K_2(T) = OC + IHC + CD - IE_2 \quad (12)
\]

Case-3: \( N \geq T \)

The interest earned per unit time is,

\[
IE_3 = PL_e R(M - N) \quad (13)
\]

Similar as case 2, total interest charge = 0

Hence, the total cost per unit time is,

\[
K_3(T) = OC + IHC + CD - IE_3 \quad (14)
\]
Hence, the total relevant cost $K(T)$ per time unit is,

$$K(T) = \begin{cases} 
K_1(T), & \text{if } M \leq T \\
K_2(T), & \text{if } N \leq T \leq M \\
K_3(T), & \text{if } N \geq T 
\end{cases}$$

(15)

where,

$$K_1(T) = \frac{A}{T} + \frac{hR}{\theta^2 T} \left( e^{\theta T} - \theta T - 1 \right) + \frac{CR}{\theta T} \left( e^{\theta T} - \theta T - 1 \right) + \frac{CL_e R}{\theta^2 T} \left( e^{\theta(T-M)} - \theta(T-M) - 1 \right) - \frac{PI_e R(M^2 - N^2)}{2T}$$

(16)

and

$$K_2(T) = \frac{A}{T} + hRT + \frac{CR \theta T}{2} + \frac{CL_e R}{2} \left( \theta^2(T-M)^2 \right) - \frac{PI_e R(M^2 - N^2)}{2T}$$

(17)

and

$$K_3(T) = \frac{A}{T} + hRT + \frac{CR \theta T}{2} - \frac{PI_e R(M-N)}{2T}$$

(18)

Corresponding to the cost $K_1(T)$, optimal cycle time $T_1$ is found out by taking the first-order and second-order derivative of equation (16) with respect to $T$. We obtain,

$$\frac{dK_1(T)}{dT} = \frac{A}{T^2} + \frac{hR}{2} + \frac{CR \theta}{2} + \frac{CL_e R}{2} \left( 1 - \frac{M^2}{T^2} \right) + \frac{PI_e R(M^2 - N^2)}{2T^2}$$

(19)

and

$$\frac{d^2 K_1(T)}{dT^2} = \frac{2A}{T^3} + \frac{CL_e R M^2}{T^3} - \frac{PI_e R(M^2 - N^2)}{T^3} > 0$$

(20)

So, for the optimal cycle time $T_1$,

$$\frac{dK_1(T)}{dT} = 0$$

or

$$\frac{A}{T^2} + \frac{hR}{2} + \frac{CR \theta}{2} + \frac{CL_e R}{2} \left( 1 - \frac{M^2}{T^2} \right) + \frac{PI_e R(M^2 - N^2)}{2T^2} = 0$$
or 

\[ T = \sqrt{\frac{2A + RCi_c M^2 + RPI_c \left(N^2 - M^2\right)}{R\left[h + C(\theta + I_c)\right]}} \]

Therefore, corresponding to cost \( K_1(T) \), the optimal cycle time \( T_1 \) is given as below:

\[ T_1 = \sqrt{\frac{2A + RCi_c M^2 + RPI_c \left(N^2 - M^2\right)}{R\left[h + C(\theta + I_c)\right]}} \tag{21} \]

Now, corresponding to the cost \( K_2(T) \), optimal cycle time \( T_2 \) is found out by taking the first-order and second-order derivative of equation (17) with respect to \( T \). We obtain,

\[ \frac{dK_2(T)}{dT} = -\frac{A}{T^2} + \frac{hR}{2} + \frac{CR\theta}{2} + \frac{PI_c R}{2} - \frac{PI_c R N^2}{2T^2} \tag{22} \]

and

\[ \frac{d^2K_2(T)}{dT^2} = \frac{2A}{T^3} + \frac{PI_c R N^2}{T^3} > 0 \tag{23} \]

So, for the optimal cycle time \( T_2 \),

\[ \frac{dK_2(T)}{dT} = 0 \]

or

\[ -\frac{A}{T^2} + \frac{hR}{2} + \frac{CR\theta}{2} + \frac{PI_c R}{2} = 0 \]

or

\[ T = \sqrt{\frac{2A + PI_c R N^2}{R\left[h + PI_c + C\theta\right]}} \]

Therefore, corresponding to cost \( K_2(T) \), the optimal cycle time \( T = T_2 \) is given as below:

\[ T_2 = \sqrt{\frac{2A + RPI_c N^2}{R\left[h + PI_c + C\theta\right]}} \tag{24} \]

Now, corresponding to the cost \( K_3(T) \), optimal cycle time \( T = T_3 \) is found out by taking the first-order and second-order derivative of equation (18) with respect to \( T \). We obtain,

\[ \frac{dK_3(T)}{dT} = -\frac{A}{T^2} + \frac{hR}{2} + \frac{CR\theta}{2} \tag{25} \]

and

\[ \frac{d^2K_3(T)}{dT^2} = \frac{2A}{T^3} > 0 \tag{26} \]
So, for the optimal cycle time $T_3$,

$$\frac{dK_3(T)}{dT} = 0$$

or

$$-\frac{A}{T^2} + \frac{hR}{2} + \frac{CR\theta}{2} = 0$$

or

$$T = \sqrt{\frac{2A}{R(h+C\theta)}}$$

Therefore, corresponding to cost $K_3(T)$, the optimal cycle time $T = T_3$ is given as below:

$$T_3 = \sqrt{\frac{2A}{R(h+C\theta)}} \quad \text{(27)}$$

4. Fuzzy Methodology

Here $A, R$ and $P, \theta$ are not known precisely and let $A, R$ and $P, \theta$ be defined by triangular fuzzy numbers such that

$$\tilde{A} = [a_1, a_2, a_3], \quad \tilde{R} = [r_1, r_2, r_3] \quad \text{and} \quad \tilde{P} = [p_1, p_2, p_3], \quad \tilde{\theta} = [\theta_1, \theta_2, \theta_3]$$

where $(a_1 < a_2 < a_3), (r_1 < r_2 < r_3)$ and $(p_1 < p_2 < p_3), \quad (\theta_1 < \theta_2 < \theta_3)$ based on subjective judgments. We apply arithmetic operators on fuzzy quantities and then de-fuzzify the same to convert them into crisp output. The membership functions for $\tilde{A}, \tilde{R}$ and $\tilde{P}, \tilde{\theta}$ are defined as follows:

$$\mu_{\tilde{A}}(\tilde{A}) = \begin{cases} 0, & \text{if } A < a_1 \\ \frac{A-a_1}{a_2-a_1}, & \text{if } a_1 \leq A < a_2 \\ \frac{a_3-A}{a_3-a_2}, & \text{if } a_2 \leq A < a_3 \\ 0, & \text{if } A \geq a_3 \end{cases}$$

(I)

$$\mu_{\tilde{R}}(\tilde{R}) = \begin{cases} 0, & \text{if } R < r_1 \\ \frac{R-r_1}{r_2-r_1}, & \text{if } r_1 \leq R < r_2 \\ \frac{r_3-R}{r_3-r_2}, & \text{if } r_2 \leq R < r_3 \\ 0, & \text{if } R \geq r_3 \end{cases}$$

(II)

$$\mu_{\tilde{P}}(\tilde{P}) = \begin{cases} 0, & \text{if } P < p_1 \\ \frac{P-p_1}{p_2-p_1}, & \text{if } p_1 \leq P < p_2 \\ \frac{p_3-P}{p_3-p_2}, & \text{if } p_2 \leq P < p_3 \\ 0, & \text{if } P \geq p_3 \end{cases}$$

(III)

$$\mu_{\tilde{\theta}}(\tilde{\theta}) = \begin{cases} 0, & \text{if } \theta < \theta_1 \\ \frac{\theta-\theta_1}{\theta_2-\theta_1}, & \text{if } \theta_1 \leq \theta < \theta_2 \\ \frac{\theta_3-\theta}{\theta_3-\theta_2}, & \text{if } \theta_2 \leq \theta < \theta_3 \\ 0, & \text{if } \theta \geq \theta_3 \end{cases}$$

(IV)
Now using the concept of ‘α−cut’ method, we see from (I), (II), (III) and (IV)

\[ ^\alpha A = [\alpha(a_2 - a_1) + a_1, a_3 - \alpha(a_3 - a_2)] \] for \( \alpha \in [0,1] \),

\[ ^\alpha R = [\alpha(r_2 - r_1) + r_1, r_3 - \alpha(r_3 - r_2)] \] for \( \alpha \in [0,1] \),

\[ ^\alpha P = [\alpha(p_2 - p_1) + p_1, p_3 - \alpha(p_3 - p_2)] \] for \( \alpha \in [0,1] \),

\[ ^\alpha \theta = [\alpha(\theta_2 - \theta_1) + \theta_1, \theta_3 - \alpha(\theta_3 - \theta_2)] \] for \( \alpha \in [0,1] \).

Then these fuzzy quantities are de-fuzzified to a crisp value by the ‘center of gravity’ method.

5. Fuzzy Inventory model

In the fuzzy model, we assume that the demand rate, ordering cost and selling price and deterioration rate are fuzzy numbers and denoted by \( \tilde{R}, \tilde{A} \) and \( \tilde{P}, \tilde{\theta} \) respectively. Here, we assume that \( \tilde{R} = (r_1, r_2, r_3), \tilde{A} = (a_1, a_2, a_3) \) and \( \tilde{P} = (p_1, p_2, p_3), \tilde{\theta} = (\theta_1, \theta_2, \theta_3) \) are non-negative triangular fuzzy numbers.

5.1 Derivation of \( \tilde{K}_1(T), \tilde{K}_2(T) \) and \( \tilde{K}_3(T) \)

The fuzzy annual total relevant cost can be expressed as,

\[
\tilde{K}(T) = \begin{cases} 
\tilde{K}_1(T), & \text{if } M \leq T \\
\tilde{K}_2(T), & \text{if } M \leq T \leq M \\
\tilde{K}_3(T), & \text{if } M \geq T 
\end{cases}
\] (28)

where

\[
\tilde{K}_1(T) = X_{11} \tilde{A} + X_{12} \tilde{R} + X_{13} \tilde{\theta} + X_{14} \tilde{P} 
\] (29)

\[
\tilde{K}_2(T) = X_{21} \tilde{A} + X_{22} \tilde{R} + X_{23} \tilde{\theta} + X_{24} \tilde{P} 
\] (30)

\[
\tilde{K}_3(T) = X_{31} \tilde{A} + X_{32} \tilde{R} + X_{33} \tilde{\theta} + X_{34} \tilde{P} 
\] (31)

where \( X_{11} = X_{21} = X_{31} = \frac{1}{T} \)

\[
X_{12} = \frac{hT}{2} + \frac{C\alpha}{2} \left[ T + \frac{M^2}{T} - 2M \right] 
\]

\[
X_{13} = X_{23} = X_{33} = \frac{CT}{2} 
\]
\[ X_{14} = -\frac{I_x(M^2 - N^2)}{2T} \]

\[ X_{22} = X_{32} = \frac{bT}{2} \]

\[ X_{24} = I_x \left( \frac{T}{2} + \frac{N^2}{2T} - M \right) \]

\[ X_{34} = -I_x(M - N). \]

Then using the concept of 'α-cut' method for \( \tilde{K}_1(T) \) and we get,

\[ \alpha \tilde{K}_1(T) = \alpha \left[ X_{11} \tilde{A} + X_{12} \tilde{R} + X_{13} \tilde{\theta} + X_{14} \tilde{R} \right] \]

\[ = \left[ X_{11} (\alpha(a_2 - a_1) + a_1) + X_{12} (\alpha(r_2 - r_1) + r_1) + X_{13} (\alpha(\theta_2 - \theta_1) + \theta_1) + \right. \]

\[ X_{14} (\alpha(p_2 - p_1) + p_1) + p_1], \]

\[ = \left[ X_{11} (a_3 - \alpha(a_3 - a_2)) + X_{12} (r_3 - \alpha(r_3 - r_2)) + X_{13} (\alpha(\theta_3 - \theta_2)), \right. \]

\[ X_{14} (p_3 - \alpha(p_3 - p_2)) [r_3 - \alpha(r_3 - r_2)] \]

\[ = \left[ \begin{array}{c}
X_{11} a_4 + X_{12} r_1 + X_{13} r_1 \theta_1 + X_{14} p_1 r_1, \\
X_{11} a_2 + X_{12} r_2 + X_{13} r_2 \theta_2 + X_{14} p_2 r_2, \\
X_{11} a_3 + X_{12} r_3 + X_{13} r_3 \theta_3 + X_{14} p_3 r_3,
\end{array} \right] \]

\[ = [k_{11}, k_{12}, k_{13}] \]

We put \( \alpha = 0 \) and \( \alpha = 1 \) in (32) and obtain an approximate triangular fuzzy number for \( \tilde{K}_1(T) \) as below:

\[ \tilde{K}_1(T) = \left[ \begin{array}{c}
X_{11} a_4 + X_{12} r_1 + X_{13} r_1 \theta_1 + X_{14} p_1 r_1, \\
X_{11} a_2 + X_{12} r_2 + X_{13} r_2 \theta_2 + X_{14} p_2 r_2, \\
X_{11} a_3 + X_{12} r_3 + X_{13} r_3 \theta_3 + X_{14} p_3 r_3,
\end{array} \right] \]

\[ = [k_{11}, k_{12}, k_{13}] \]

where \( \begin{align*}
k_{11} &= X_{11} a_1 + X_{12} r_1 + X_{13} r_1 \theta_1 + X_{14} p_1 r_1, \\
k_{12} &= X_{11} a_2 + X_{12} r_2 + X_{13} r_2 \theta_2 + X_{14} p_2 r_2, \\
k_{13} &= X_{11} a_3 + X_{12} r_3 + X_{13} r_3 \theta_3 + X_{14} p_3 r_3.
\end{align*} \)

Thus, the membership functions for \( \tilde{K}_1(T) \) is given as:

\[ \mu_{\tilde{K}_1}(K_1) = \begin{cases} 
0, & i \notin K_1 < k_{11} \\
\frac{K_1 - k_{11}}{k_{12} - k_{11}}, & k_{11} \leq K_1 < k_{12} \\
\frac{k_{13} - K_1}{k_{13} - k_{12}}, & k_{12} \leq K_1 < k_{13} \\
0, & i \notin K_1 \geq k_{13}
\end{cases} \]
\( \tilde{K}_1(T) \) is de-fuzzified to a crisp value by the ‘centre of gravity’ method. The de-fuzzified \( \tilde{K}_1(T) \) is found as,
\[
U(\tilde{K}_1(T)) = \text{defuzz}(\tilde{K}_1(T)) = \frac{\int_R \tilde{K}_1(T) \mu_{\tilde{K}_1}(\tilde{K}_1) d\tilde{K}_1(T)}{\int_R \mu_{\tilde{K}_1}(\tilde{K}_1) d\tilde{K}_1(T)} \tag{34}
\]

Similarly, using the concept of ‘\( \alpha – cut \)’ method for \( \tilde{K}_2(T) \) and we get,
\[
\alpha [\tilde{K}_2(T)] = \alpha [X_{21} \tilde{A} + X_{22} \tilde{R} + X_{23} \tilde{R} \theta + X_{24} \tilde{R} \theta]
\]
\[
= \left[ X_{21} \{\alpha(a_2 - a_1) + a_1\} + X_{22} \{\alpha(r_2 - r_1) + r_1\} + X_{23} \{\alpha(\theta_2 - \theta_1) + \theta_1\} + X_{24} \{\alpha(p_2 - p_1) + p_1\} \right] + \left[ X_{21} \{\alpha(a_3 - a_2)\} + X_{22} \{r_2 - \alpha(r_3 - r_2)\} + X_{23} \{\theta_3 - \alpha(\theta_3 - \theta_2)\} + X_{24} \{\theta_3 - \alpha(p_3 - p_2)\} \right] \tag{35}
\]

We put \( \alpha = 0 \) and \( \alpha = 1 \) in (35) and obtain an approximate triangular fuzzy number for \( \tilde{K}_2(T) \) as below:
\[
\tilde{K}_2(T) = \begin{bmatrix}
X_{21} a_1 + X_{22} r_1 + X_{23} r_1 \theta_1 + X_{24} p_1 r_1, \\
X_{21} a_2 + X_{22} r_2 + X_{23} r_2 \theta_2 + X_{24} p_2 r_2, \\
X_{21} a_2 + X_{22} r_3 + X_{23} r_3 \theta_3 + X_{24} p_3 r_3
\end{bmatrix}
\]
\[
= [k_{21}, k_{22}, k_{23}] \tag{36}
\]
where
\[
k_{21} = X_{21} a_1 + X_{22} r_1 + X_{23} r_1 \theta_1 + X_{24} p_1 r_1,
\]
\[
k_{22} = X_{21} a_2 + X_{22} r_2 + X_{23} r_2 \theta_2 + X_{24} p_2 r_2,
\]
\[
k_{23} = X_{21} a_3 + X_{22} r_3 + X_{23} r_3 \theta_3 + X_{24} p_3 r_3.
\]

Thus, the membership functions for \( \tilde{K}_2(T) \) is given as:
\[
\mu_{\tilde{K}_2}(\tilde{K}_2) = \begin{cases}
0, \text{if} \ K_2 < k_{21} \\
\frac{K_2 - k_{21}}{k_{22} - k_{21}}, \text{if} \ k_{21} \leq K_2 < k_{22} \\
\frac{K_2 - k_2}{k_{23} - k_2}, \text{if} \ k_{22} \leq K_2 < k_{23} \\
0, \text{if} \ K_2 \geq k_{23}
\end{cases} \tag{VI}
\]
\( \tilde{K}_2(T) \) is de-fuzzified to a crisp value by the ‘centre of gravity’ method. The de-fuzzified \( \tilde{K}_2(T) \) is found as,

\[
U(\tilde{K}_2(T)) = \text{defuzz}(\tilde{K}_2(T)) = \frac{\int_{R} \tilde{K}_2(T) \mu_{\tilde{K}_2}(\tilde{K}_2) d\tilde{K}_2(T)}{\int_{R} \mu_{\tilde{K}_2}(\tilde{K}_2) d\tilde{K}_2(T)} \tag{37}
\]

Similarly, using the concept of '\( \alpha - \) cut' method for \( \tilde{K}_3(T) \) and we get,

\[
\quad\quad a[\tilde{K}_3(T)] = a\left[X_{31}A + X_{32}\tilde{R} + X_{33}\tilde{R}\theta + X_{34}\tilde{P}\tilde{R}\right]
\]

\[
= \left[ X_{31}\{\alpha(a_2 - a_1) + a_1\} + X_{32}\{\alpha(r_2 - r_1) + r_1\} + X_{33}\{\alpha(r_2 - r_1) + r_1\}\{\alpha(\theta_2 - \theta_1) + \theta_1\} + \right.
\]

\[
X_{34}\{\alpha(p_2 - p_1) + p_1\}\{\alpha(r_2 - r_1) + r_1\},
\]

\[
X_{31}\{(a_3 - \alpha(a_3 - a_2)) + X_{32}\{r_3 - \alpha(r_3 - r_2)\}\} + X_{33}\{r_3 - \alpha(r_3 - r_2)\}\{\theta_3 - \alpha(\theta_3 - \theta_2)\} +
\]

\[
X_{34}\{(p_3 - \alpha(p_3 - p_2))\{r_3 - \alpha(r_3 - r_2)\}\}
\]

\( \tag{38} \)

We put \( \alpha = 0 \) and \( \alpha = 1 \) in (38) and obtain an approximate triangular fuzzy number for \( \tilde{K}_3(T) \) as below:

\[
\tilde{K}_3(T) = \left[ X_{31}a_1 + X_{32}r_1 + X_{33}r_1\theta_1 + X_{34}p_1r_1, \right.
\]

\[
X_{31}a_2 + X_{32}r_2 + X_{33}r_2\theta_2 + X_{34}p_2r_2,
\]

\[
X_{31}a_3 + X_{32}r_3 + X_{33}r_3\theta_3 + X_{34}p_3r_3 \right]
\]

\[
= [k_{31}, k_{32}, k_{33}] \tag{39}
\]

where

\[
k_{31} = X_{31}a_1 + X_{32}r_1 + X_{33}r_1\theta_1 + X_{34}p_1r_1,
\]

\[
k_{32} = X_{31}a_2 + X_{32}r_2 + X_{33}r_2\theta_2 + X_{34}p_2r_2,
\]

\[
k_{33} = X_{31}a_3 + X_{32}r_3 + X_{33}r_3\theta_3 + X_{34}p_3r_3.
\]

Thus, the membership functions for \( \tilde{K}_3(T) \) is given as:

\[
\mu_{\tilde{K}_3}(\tilde{K}_3) = \begin{cases} 
0, & \text{if } K_3 < k_{31} \\
\frac{K_3 - k_{31}}{k_{32} - k_{31}}, & \text{if } k_{31} \leq K_3 < k_{32} \\
\frac{k_{33} - K_3}{k_{33} - k_{32}}, & \text{if } k_{32} \leq K_3 < k_{33} \\
0, & \text{if } K_3 \geq k_{33}
\end{cases} \tag{VII}
\]
\( \bar{K}_3(T) \) is de-fuzzified to a crisp value by the ‘centre of gravity’ method. The de-fuzzified \( \bar{K}_3(T) \) is found as,

\[
U(\bar{K}_3(T)) = \text{defuzz}(\bar{K}_3(T)) = \frac{\int_{-\infty}^{\infty} \bar{K}_3(T) \mu_{\bar{K}_3}(T) d\bar{K}_3(T)}{\int_{-\infty}^{\infty} \mu_{\bar{K}_3}(T) d\bar{K}_3(T)}
\]

(40)

5.2 Derivation of \( \bar{T}_1, \bar{T}_2 \) and \( \bar{T}_3 \)

The fuzzy total cycle time can be expressed as,

\[
\bar{T} = \begin{cases} 
\bar{T}_1, & \text{if } M \leq T \\
\bar{T}_2, & \text{if } N \leq T \leq M \\
\bar{T}_3, & \text{if } N \geq T 
\end{cases}
\]

(41)

where

\[
\bar{T}_1 = \sqrt{\frac{2A + \bar{R}CI_c M^2 + \bar{R}P I_c N^2}{\bar{R} h + C(\bar{\theta} + I_c)}}
\]

(42)

and

\[
\bar{T}_2 = \sqrt{\frac{2A + \bar{R}P I_c N^2}{\bar{R} h + \bar{P} I_c + C\bar{\theta}}}
\]

(43)

and

\[
\bar{T}_3 = \sqrt{\frac{2A}{\bar{R}(h + C\bar{\theta})}}
\]

(44)

From inequality

\[
\begin{bmatrix}
\alpha (a_3 - a_2) + a_1 \\
\alpha (r_3 - r_2) + r_1
\end{bmatrix} \geq \begin{bmatrix}
\alpha (a_2 - a_1) + a_1 \\
r_3 - \alpha (r_3 - r_2)
\end{bmatrix}
\]

for \( \alpha \in [0,1] \).

Thus,

\[
\begin{bmatrix}
\alpha \\
\frac{A}{R}
\end{bmatrix} = \begin{bmatrix}
\alpha (a_2 - a_1) + a_1 \\
\alpha (a_3 - a_2) + a_1 \\
r_3 - \alpha (r_3 - r_2) + r_1
\end{bmatrix}
\]

Then using the concept of '\( \alpha - \text{cut} \)' method for \( \bar{T}_1 \), and we get,
\[
\alpha [\tilde{T}_i] = \alpha \left[ \sqrt{\frac{2\tilde{A} + \tilde{R}C_1 M^2 + \tilde{R}\tilde{I}_e (N^2 - M^2)}{\tilde{R} \left( h + C(\theta + I_c) \right)}} \right]
\]
\[
= \left[ \begin{array}{l}
2[\alpha(a_2 - a_1) + a_i] + CI_1 M^2 \{[\alpha(r_2 - r_1) + r_1] + I_e (N^2 - M^2)\} \{r_2 - r_1\} + r_1 \{\alpha(p_2 - p_1) + p_1\} \\
\{r_2 - \alpha(r_2 - r_1)\} \{h + C(\theta_2 - \theta_1 + I_e)\}\end{array} \right]
\]
\[
\sqrt{\left[ \begin{array}{l}
2[\alpha(a_3 - a_2)] + CI_1 M^2 \{r_2 - \alpha(r_3 - r_2)\} + I_e (N^2 - M^2) \{r_2 - \alpha(r_3 - r_2)\} \{p_3 - \alpha(p_3 - p_2)\} \\
\{\alpha(r_3 - r_2)\} + r_2 \{h + C(\theta_3 - \theta_2 + I_e)\}\end{array} \right]}
\]

(45)

We put \( \alpha = 0 \) and \( \alpha = 1 \) in (45) and obtain an approximate triangular fuzzy number for \( \tilde{T}_i \) as below:

\[
\tilde{T}_i = \left[ \begin{array}{l}
\frac{2a_i + CI_1 M^2 r_i + I_e (N^2 - M^2) r_i p_1}{r_i \{h + C(\theta_1 + I_c)\}} \\
\frac{2a_2 + CI_1 M^2 r_2 + I_e (N^2 - M^2) r_2 p_2}{r_2 \{h + C(\theta_2 + I_c)\}} \\
\frac{2a_3 + CI_1 M^2 r_3 + I_e (N^2 - M^2) r_3 p_3}{r_3 \{h + C(\theta_3 + I_c)\}}
\end{array} \right]
\]

(46)

where

\[
t_{11} = \frac{2a_i + CI_1 M^2 r_i + I_e (N^2 - M^2) r_i p_1}{r_i \{h + C(\theta_1 + I_c)\}}
\]
\[
t_{12} = \frac{2a_2 + CI_1 M^2 r_2 + I_e (N^2 - M^2) r_2 p_2}{r_2 \{h + C(\theta_2 + I_c)\}}
\]
\[
t_{13} = \frac{2a_3 + CI_1 M^2 r_3 + I_e (N^2 - M^2) r_3 p_3}{r_3 \{h + C(\theta_3 + I_c)\}}
\]

Thus, the membership functions for \( \tilde{T}_i \) is given as:

\[
\mu_{\tilde{T}_i}(\tilde{T}_i) = \begin{cases} 
0, & \text{if } T_i < t_{11} \\
\frac{T_i - t_{11}}{t_{12} - t_{11}}, & \text{if } t_{11} \leq T_i < t_{12} \\
\frac{T_i - t_{12}}{t_{13} - t_{12}}, & \text{if } t_{12} \leq T_i < t_{13} \\
0, & \text{if } T_i \geq t_{13}
\end{cases}
\]

(VIII)
\[ U(\tilde{T}_1) = \text{defuzz}(\tilde{T}_1) = \frac{\int_{\tilde{T}_1} \mu_{\tilde{T}_1}(\tilde{T}_1) d\tilde{T}_1}{\int_{\tilde{T}_1} \mu_{\tilde{T}_1}(\tilde{T}_1) d\tilde{T}_1} \]  

(47)

Similarly, using the concept of \( \alpha - \text{cut} \) method for \( \tilde{T}_2 \) and we get,

\[ \alpha[\tilde{T}_2] = \left[ \begin{array}{c} 2\{\alpha(a_2 - a_1) + a_1\} + I_e N^2 \{\alpha(r_2 - r_1) + r_1\} \{\alpha(p_2 - p_1) + p_1\} \\ \sqrt{\{r_3 - \alpha(r_3 - r_2)\} \{h + C[\theta_3 - \alpha(\theta_3 - \theta_2)] + \{p_3 - \alpha(p_3 - p_2)\} I_e\}} \\ 2\{\alpha(a_2 - a_1) + a_1\} + I_e N^2 \{r_3 - \alpha(r_3 - r_2)\} \{p_3 - \alpha(p_3 - p_2)\} \\ \sqrt{\{\alpha(r_2 - r_1) + r_1\} \{h + C[\alpha(\theta_2 - \theta_1) + \theta_1\} + \{\alpha(p_2 - p_1) + p_1\} I_e\}} \end{array} \right] \]  

(48)

We put \( \alpha = 0 \) and \( \alpha = 1 \) in (48) and obtain an approximate triangular fuzzy number for \( \tilde{T}_2 \) as below:

\[ \tilde{T}_2 = \left[ t_{21}, t_{22}, t_{23} \right] \]  

(49)

where

\[ t_{21} = \frac{2a_1 + I_e N^2 r_1 p_1}{r_3(h + C\theta_3 + p_1 I_e)}, \quad t_{22} = \frac{2a_2 + I_e N^2 r_2 p_2}{r_2(h + C\theta_2 + p_2 I_e)}, \quad t_{23} = \frac{2a_3 + I_e N^2 r_3 p_3}{r_1(h + C\theta_1 + p_1 I_e)} \]

Thus, the membership functions for \( \tilde{T}_2 \) is given as:

\[ \mu_{\tilde{T}_2}(\tilde{T}_2) = \begin{cases} 0, & \text{if } \tilde{T}_2 < t_{21} \\ \frac{T_2 - t_{21}}{t_{22} - t_{21}}, & \text{if } t_{21} \leq \tilde{T}_2 < t_{22} \\ \frac{t_{23} - \tilde{T}_2}{t_{23} - t_{22}}, & \text{if } t_{22} \leq \tilde{T}_2 < t_{23} \\ 0, & \text{if } \tilde{T}_2 \geq t_{23} \end{cases} \]  

(IX)
\( \tilde{T}_2 \) is de-fuzzified to a crisp value by the ‘centre of gravity’ method. The de-fuzzified \( \tilde{T}_2 \) is found as,

\[
U(\tilde{T}_2) = \text{defuzz} (\tilde{T}_2) = \int_{-\infty}^{\infty} \mu_{\tilde{T}_2}(\tilde{T}_2) d\tilde{T}_2 = \int_{-\infty}^{\infty} \mu_{\tilde{T}_2}(\tilde{T}_2) d\tilde{T}_2
\]

(50)

Similarly, using the concept of 'α - cut' method for \( \tilde{T}_3 \) and we get,

\[
\alpha [\tilde{T}_3] = \alpha \left[ \sqrt{\frac{2A}{R(h + C \theta)}} \right]
\]

\[
= \left[ \frac{2\alpha(a_2 - a_1) + a_1}{\sqrt{r_3 - \alpha(r_3 - r_2)(h + Cr_3 - \alpha(\theta_3 - \theta_1))}} \right] \left[ \frac{2\alpha(a_3 - a_2) + a_2}{\sqrt{r_2 - \alpha(r_2 - r_1)(h + Cr_2 - \alpha(\theta_2 - \theta_1) + \theta_1)}} \right] \left[ \frac{2\alpha(a_3 - a_2) + a_3}{\sqrt{r_1 - \alpha(r_1 - r_1)(h + Cr_1)}} \right]
\]

(51)

We put \( \alpha = 0 \) and \( \alpha = 1 \) in (51) and obtain an approximate triangular fuzzy number for \( \tilde{T}_3 \) as below:

\[
\tilde{T}_3 = \left[ \frac{2a_1}{r_3(h + C\theta_3)}, \frac{2a_2}{r_2(h + C\theta_2)}, \frac{2a_3}{r_1(h + C\theta_1)} \right] = [t_{31}, t_{32}, t_{33}]
\]

(52)

where \( t_{31} = \frac{2a_1}{r_3(h + C\theta_3)}, t_{32} = \frac{2a_2}{r_2(h + C\theta_2)}, t_{33} = \frac{2a_3}{r_1(h + C\theta_1)} \).

Thus, the membership functions for \( \tilde{T}_3 \) is given as:

\[
\mu_{\tilde{T}_3}(\tilde{T}_3) = \begin{cases} 
0, & \text{if } T_3 < t_{31} \\
\frac{T_3 - t_{31}}{t_{32} - t_{31}}, & \text{if } t_{31} \leq T_3 < t_{32} \\
\frac{t_{33} - T_3}{t_{33} - t_{32}}, & \text{if } t_{32} \leq T_3 < t_{33} \\
0, & \text{if } T_3 \geq t_{33}
\end{cases}
\]

(X)
\( \tilde{T}_3 \) is de-fuzzified to a crisp value by the ‘centre of gravity’ method. The de-fuzzified \( \tilde{T}_3 \) is found as,

\[
U(\tilde{T}_3) = \text{defuzz}(\tilde{T}_3) = \frac{\int_{\tilde{T}_3} T \mu_{\tilde{T}_3}(T) d\tilde{T}_3}{\int_{\tilde{T}_3} \mu_{\tilde{T}_3}(T) d\tilde{T}_3}
\]  

(53)

### 6. Numerical example

To illustrate the results obtained in this paper, let us apply the proposed method to solve the following numerical example. We use MAPLE (version 11) software for all these calculations.

Let \( \tilde{A} = (48,50,52) \) \textit{order} , \( \tilde{R} = (480,500,520) \) \textit{units/year} , \( \tilde{\theta} = (0.10,0.20,0.30) \) be triangular fuzzy numbers.

\[ h = $5/unit/year , I_e = $0.12/unit/year , I_c = $0.15/unit/year , C = $50/unit , M = 0.1/year . \]

To find the optimal cycle time (between \( T_1 \), \( T_2 \) and \( T_3 \)) and total optimal cost (between \( K_1(T_1) \), \( K_2(T_2) \) and \( K_3(T_3) \)) for different values of \( N \), we use the following algorithm. The steps are as follows:

**Step-1:** Compute \( T_1 \), \( T_2 \) and \( T_3 \) by solving eq. (47), (50) and (53) respectively.

**Step-2:** If \( M \leq T_1 \), then compute \( K_1(T_1) \), otherwise go to step-3.

**Step-3:** If \( N \leq T_2 \leq M \), then compute \( K_2(T_2) \), otherwise go to step-4.

**Step-4:** If \( N \geq T_3 \), then compute \( K_3(T_3) \).

**Step-5:** Find corresponding cycle time and total optimal cost.
In the above table-1, red color values shows the corresponding optimal cycle time and optimal cost for different combinations of $P$ and $N$. 

<table>
<thead>
<tr>
<th>$\tilde{P}$</th>
<th>$N = 0.02\text{ year}$</th>
<th>$N = 0.05\text{ year}$</th>
<th>$N = 0.08\text{ year}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(118,120,122)$</td>
<td>$T_1$ 0.0767</td>
<td>0.0848</td>
<td>0.0982</td>
</tr>
<tr>
<td></td>
<td>$T_2$ 0.0820</td>
<td>0.0878</td>
<td>0.0978</td>
</tr>
<tr>
<td></td>
<td>$T_3$ 0.1179</td>
<td>0.1179</td>
<td>0.1179</td>
</tr>
<tr>
<td></td>
<td>$K_1(T_1)$ 617.67</td>
<td>720.37</td>
<td>889.21</td>
</tr>
<tr>
<td></td>
<td>$K_2(T_2)$ 633.42</td>
<td>727.91</td>
<td>888.74</td>
</tr>
<tr>
<td></td>
<td>$K_3(T_3)$ 473.90</td>
<td>685.60</td>
<td>897.21</td>
</tr>
<tr>
<td>$(158,160,162)$</td>
<td>$T_1$ 0.0624</td>
<td>0.0751</td>
<td>0.0943</td>
</tr>
<tr>
<td></td>
<td>$T_2$ 0.0763</td>
<td>0.0834</td>
<td>0.0952</td>
</tr>
<tr>
<td></td>
<td>$T_3$ 0.1179</td>
<td>0.1179</td>
<td>0.1179</td>
</tr>
<tr>
<td></td>
<td>$K_1(T_1)$ 435.31</td>
<td>596.22</td>
<td>839.53</td>
</tr>
<tr>
<td></td>
<td>$K_2(T_2)$ 489.40</td>
<td>621.71</td>
<td>841.53</td>
</tr>
<tr>
<td></td>
<td>$K_3(T_3)$ 284.57</td>
<td>567.32</td>
<td>849.91</td>
</tr>
<tr>
<td>$(180,200,220)$</td>
<td>$T_1$ 0.0415</td>
<td>0.0624</td>
<td>0.0897</td>
</tr>
<tr>
<td></td>
<td>$T_2$ 0.0723</td>
<td>0.0808</td>
<td>0.0945</td>
</tr>
<tr>
<td></td>
<td>$T_3$ 0.1179</td>
<td>0.1179</td>
<td>0.1179</td>
</tr>
<tr>
<td></td>
<td>$K_1(T_1)$ 220.79</td>
<td>471.30</td>
<td>794.53</td>
</tr>
<tr>
<td></td>
<td>$K_2(T_2)$ 361.92</td>
<td>530.32</td>
<td>803.16</td>
</tr>
<tr>
<td></td>
<td>$K_3(T_3)$ 120.87</td>
<td>466.34</td>
<td>810.05</td>
</tr>
</tbody>
</table>
The following inferences can be made based on Table-1.

- When $N$ is increasing, the optimal cycle time and optimal cost for the retailer is also increasing. It implies that the retailer will order more quantity to get more interest earned offered by the supplier to compensate the loss of interest earned from longer trade credit period to his/her customer.
- When $P$ is increasing, the optimal time and optimal cost is decreasing. So we observed that the retailer will not order more quantity to take the benefits of the delay payments more frequently when the larger the difference between the unit selling price and the unit purchasing price.

7. Conclusion

In this paper, we have developed an EOQ model for deteriorating items under two-levels of trade credit policy in the fuzzy sense. The demand rate, ordering cost and selling price, deterioration rate are assumed as triangular fuzzy numbers. The fuzzy total variable cost and fuzzy cycle time are derived. By the centre of gravity method, we de-fuzzified the fuzzy total cost and fuzzy cycle time. Numerical example reveals that a higher value of the permissible delay period $N$ increases the total cycle time and total cost of the retailer.

Acknowledgements: The authors like to thank anonymous reviewers for their constructive comments. The department of first author is supported by DST-FIST.

References


**Appendix**

**Fuzzy set theory:** We include a brief introduction on fuzzy set theory. More details are available with Klir et al. (2005) and Lee (2005).

**Definition-1** A fuzzy set is a set where the members are allowed to have partial membership and hence the degree of membership varies from 0 to 1. It is expressed as, $A = \{ (x, \mu_A(x)) | x \in X \}$ where $X$ is the universe of discourse and $\mu_A(x)$ is the universe of discourse and $\mu_A(x) = 0 \text{ or } 1$ i.e., $x$ is a non-member in $A$ if $\mu_A(x) = 0$, and $x$ is a member in $A$ if $\mu_A(x) = 1$.

**Definition-2** If a fuzzy set $A$ is defined on $X$, for any $\alpha \in [0,1]$, the $\alpha$-cuts $^\alpha A$ is represented by the following crisp set,

Strong $\alpha$-cuts: $^\alpha A = \{ x \in X | \mu_A(x) > \alpha \}$, $\alpha \in [0,1]$  

Weak $\alpha$-cuts: $^\alpha A = \{ x \in X | \mu_A(x) \geq \alpha \}$, $\alpha \in [0,1]$.

**Fuzzy Arithmetic Operations:** We define fuzzy arithmetic operations on fuzzy numbers in terms of the $\alpha$-cuts. Let, $A$ and $B$ are two fuzzy sets and if ‘*’ denotes any of the four basic arithmetic operations $(+, -, *, /)$ then a fuzzy set $Z = (A * B)$ and $Z \in R$, can be defined as, $^\alpha (A * B) = ^\alpha A * ^\alpha B$ such that $\forall \alpha \in (0,1]$. 
However, if ‘∗’ is a division operator, then $^\alpha (A * B) = ^\alpha A * ^\alpha B$, such that $\forall \alpha \in (0,1]$ and $0 \in ^\alpha B$.

**Theorem-1 (First decomposition theorem)**

For every $A \in X$,

$$A = \bigcup_{\alpha \in (0,1]} A, \text{where } ^\alpha A(x) = ^\alpha A(x)$$

From first decomposition theorem, if $Z = (A * B)$ and $Z \in R$,

$$(A * B) = \bigcup_{\alpha \in (0,1]} (A * B).$$

Since $^\alpha (A * B)$ is a closed interval for each $\alpha \in (0,1]$ with both A and B fuzzy, $(A * B)$ is also a fuzzy number.

**Definition-3** For the de-fuzzification of a fuzzy set to a crisp value, ‘centre of gravity’ or ‘moment method’ is a popular and efficient approach. $\tilde{A}(x)$ is converted to a crisp value by the following operation,

$$A = \text{defuzz}(\tilde{A}) = \frac{\int_{R} A \mu_{\tilde{A}}(x)dx}{\int_{R} \mu_{\tilde{A}}(x)dx}$$

These three cases are graphed in figure-1, 2 and 3 respectively.

![Figure-1 The total accumulation of interest earned when $M \leq T$](image-url)
Figure-2 The total accumulation of interest earned when $N \leq T \leq M$

Figure-3 The total accumulation of interest earned when $N \geq T$