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Mathematical Study of Anarchy Control in Two – dimensional Prey – Predator Model

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Abstract

Content in this paper depicts probable suppression of anarchy on x-y plane and the corresponding graphs, on attaining equilibrium position, are called trajectories. This model represents the competition between populations of two species and study including three types of interactions amongst them eventually establishes that growth rates of population are non-sympathetic in direction. The situation when the growth rates show sympathetic movement in decreasing pattern; we call this situation as 'competition' and 'mutualism' if it exhibits sympathetic movement in increasing pattern.

Keywords: The Prey – predator Model, Controlling of anarchy, trajectory.
AMS Subject Classification: 92B05

Introduction:

In survey carried out in 1997, it has been recorded that in the last forty years the presence of anarchy in various fields (Physical, Chemical, Biological, or Economical) has been extensively demonstrated. The basic characteristics of the hapless system, which are debauched nonlinear dynamic systems, defined as a set of coupled differential equations or as a map is the extreme sensitivity to small discrepancies in their preliminary situations [4]. This characteristic has been considered to be dreadful property and for many years it was generally believed that helter – skelter behaviors are uncontrollable. In 1997, results of the studies on the same lines have demonstrated that even this property permits the use of small perturbation to control phase trajectories of these systems.

There are three types of interactions as shown below;

(i) If the progress rate of one population reduces and that of the other one improves then both the populations are said to be in a predator–prey position.
(ii) If the progress rate of each population reduces then it is said to be race.
(iii) If progress rate each population is improved then it is called mutualism. [7].

A mathematical model is an explanation of procedure describing mathematical ideas and language.

The view of Mathematical Modeling is defined in different ways in curriculum of mathematics. In mathematical research education one may invent and change perceptions and attitudes on Mathematical Modeling.

Mathematical modeling (the procedure of interpreting between the real world and mathematics in both ways) is one of the topics in mathematics learning that has been thought and conducted most passionately during the last few eras.

The Prey – predator Model

We consider a model in which there are two species of animals in that one lives on other killing.

The species which kills the other is called the predator species or killer species and the species getting kill is called the prey –species or killed species.

Here the contact between Prey and Predator harmful to Prey and useful to Predator also for separation between Prey and Predator harmful to Predator and useful to Prey.

Let $x(t)$ = Population of prey species at timet and $y(t)$ = Population of predator species at timet

In the absence of Predator, the population of prey species increases and this growth rate is proportion to the number of prey species[2].

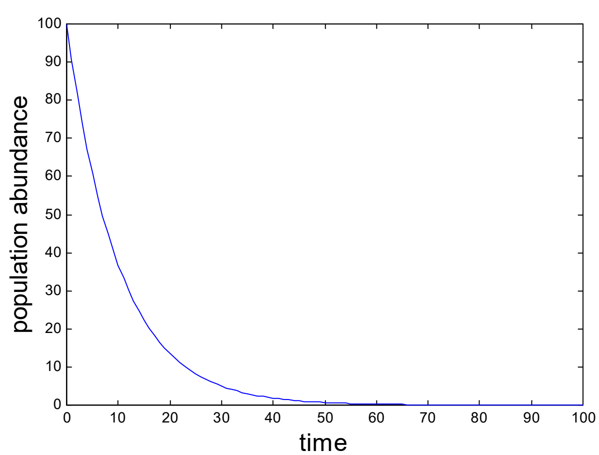
$$i.e.\ \frac{dx}{dt} \propto x \ (1)$$

In the absence of Prey, the population of Predator species decreases and this decay rate is proportion to the number of Predator species.

$$i.e.\ \frac{dy}{dt} \propto -y \ \ (2)$$

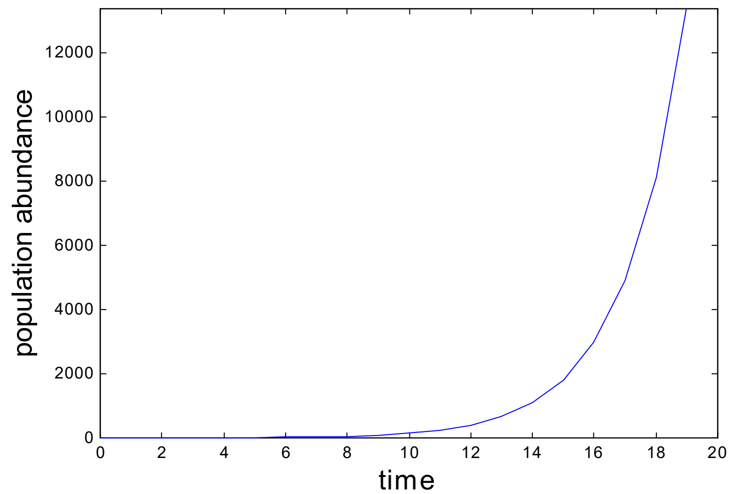
In the presence of Predator, the population of Prey species decreases and this decay rate is proportion to the product of prey and Predator species at that time.

$$i.e.\ \frac{dx}{dt} \propto -xy \ (3)$$



In the presence of Prey, the population of Predator species increases and this growth rate is proportion to the product of prey and Predator species at that time.

$$i.e.\ \frac{dy}{dt} \propto xy\ (4)$$



By assumption $(1)\ \ and\ \ (3)$

$$\frac{dx}{dt} = ax - bxy(5)$$

Also by assumption $(2)\ \ and\ \ (4)$

$$\frac{dy}{dt} = -cy + dxy(6)$$

For the equilibrium position $\frac{dx}{dt} = 0 \Rightarrow ax - bxy = 0 \Rightarrow x = 0\ \ or\ \ y = \frac{a}{b}$

$$\frac{dy}{dt} = 0 \Rightarrow cy + dxy = 0 \Rightarrow y = 0\ \ or\ \ x = \frac{c}{d}$$

Now we have two equilibrium positions $(0{,}0)\ \ and\ \ \left(\frac{c}{d}, \frac{a}{b}\right)$

So that if the initial population size is $\left(\frac{c}{d}, \frac{a}{b}\right)$ then the population remains constant

Now from $(5)\ \ and\ \ (6)$

If $x < \frac{c}{d} \Rightarrow dx - c < 0 \Rightarrow \frac{dy}{dt} < 0 \Rightarrow y\ is\ decreases$

If $y < \frac{a}{b} \Rightarrow by - a < 0 \Rightarrow \frac{dx}{dt} > 0 \Rightarrow x\ is\ increases$

If this increases value is less then $\frac{c}{d}$ as above process again x is increase and attends $\frac{c}{d}$

If $y > \frac{a}{b}$thena $- by < 0 \therefore x\ is\ decrease$ If $x > \frac{c}{d} \therefore y\ is\ decreases$

If this decreased value is again greater than by above process again $y - decrease$ by continue this process y must attends the value $\frac{a}{b}$

Similarly we can prove for the $x > \frac{c}{d}\ \ and\ \ y > \frac{a}{b}$

In all the cases the equilibrium condition is re – establish so that $\left(\frac{c}{d}, \frac{a}{b}\right)$ is stable condition.

Now consider the equilibrium point $(0{,}0)$

Suppose at time $t\ , x > 0\ \ \ \therefore\ \ \frac{dx}{dt} > 0$ then x increases and any time y become zero hence we cannot return at the original position$(0{,}0)$.

Hence the equilibrium point $(0,0)$ is unstable.

Now by (5) and $(6)\,\frac{dy}{dt} = -cy + dxy$, $\frac{dx}{dt} = ax - bxy$

$$\therefore \frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt} = \frac{-cy + dxy}{ax - bxy} = \frac{y(dx - c)}{x(a - by)}$$

$$\therefore \frac{a - by}{y}\, dy = \frac{dx - c}{x}\, dx$$

$$\therefore \left(\frac{a}{y} - b\right) dy = \left(d - \frac{c}{x}\right) dx$$

$$\therefore a\, logy - by = dx - c\, logx + c_1$$

$$\therefore \log y^a + \log x^c = dx + by + c_1$$

At $t = 0\,, x(0) = x_0$ and $y(0) = y_0$

$$\therefore\ c_1 = \log {y_0}^a + \log {x_0}^c - dx_0 - by_0$$

$$\therefore \log y^a + \log x^c = dx + by + \log {y_0}^a + \log {x_0}^c - dx_0 - by_0$$

$$\therefore \log\left(\frac{y}{y_0}\right)^a + \log\left(\frac{x}{x_0}\right)^c = d(x - x_0) + b(y - y_0) (7)$$

The graph of equation (7) in $xy - plane$ is called trajectories.

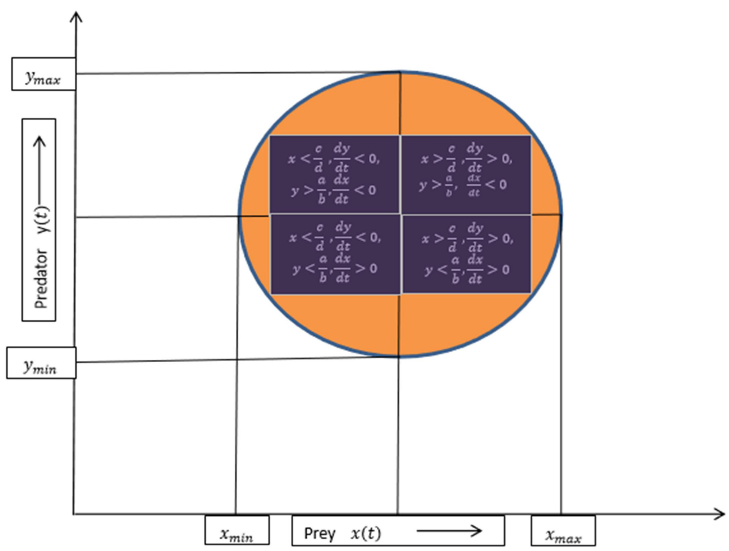


Figure:1 Trajectories of predator– prey for (7)

Conclusion:

In this paper, a two -dimensional isolated predator - prey model is consideredand gained equilibrium points. Steady conditions are found and the results are explained with appropriate parameter values. For example, food resources of prey populations are limited and not only by predation and no predator can eat infinite quantities of prey. Many other examples of repeated relationships between predator and prey populations have been established in the laboratory or experiential in nature, but in general these are healthier fitting by models including terms that denote deep capacity.

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