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Preservation technology investment for non-instantaneous deteriorating items in vendor managed inventory system

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Abstract

Present study construct total cost for traditional supply chain model and vender managed inventory supply chain model for non-instantaneous deteriorating products when preservation technology is applied where original deterioration rate is constant. Demand is constant before deterioration starts and decreases exponentially during deterioration. Objective of this study is to optimize replenishment policy and to show effect of preservation technology investment on decision variable. Numerical example illustrates features of the model. Sensitivity analysis is given to check liability of the model at the end of this paper.

Keywords: Vendor managed inventory (VMI), non-instantaneous deteriorating item, preservation technology.
AMS Subject Classification: 90B05

1. Introduction

Deterioration is "damage, spoilage, dryness, vaporization, etc. which may decrease the usefulness of the original product" (Bai et al., 2016). Products such as milk, vegetables, fruits etc. start losing their utility during storage. Admittedly, increased market uncertainty due to globalized economy, it is a need of hour for companies to integrate with customers through better-integrated supply chain (Hongjie et al., 2011). Therefore, managing inventory for deteriorating items has been a challenging issue in supply chain management and always been paid a due consideration in research and practice.

There is a surge of many comprehensive research works in inventory modeling for deteriorating items, followed by pioneer study of Ghare and Schrader (1963). Detailed reviews of early literature on inventory studies considering deterioration have been done by Raafat (1991), Goyal and Giri (2001), Bakker et al. (2012), Hsieh and Dye (2013), Taleizadeh and Nematollahi (2014) and Gilding (2014). In the same vein, Janssen, Claus and Sauer (2016) added a comprehensive review taking studies on deteriorating inventory models from 2012 to 2015 and used similar approach to Bakker et al. (2012) study. When cross-examining the previous literature, rare studies have considered the aspects of controlling deterioration.

Moreover, products such as fruits, vegetables, fashionable items etc. do not start deteriorating just after the storage. Such phenomenon is defined as “non-instantaneous deterioration” and ignoring this in inventory planning may result in increased inventory holding and disposing costs (Wu et al., 2006). Considering this feature, the challenge is now been transferred from retailer-manufacturer to the whole supply chain system. This non-instantaneous deterioration is comprehensively explored by many research studies such as Maihami et al. (2012), Valliathal and Uthayakumar (2013), Shah et al. (2013), Tat et al. (2013) after Wu et al. (2009)’s first modelling work.

To mitigate the risk, upstream enterprises (suppliers) act in accordance with downstream enterprises (retailers) by unifying aggregate demand with certain rules which is the core essence of vendor managed inventory (VMI) system (Hongjie et al. 2011). This helps suppliers (or vendors) to minimize chain’s total inventory cost by managing buyer’s inventory (Sadeghi et al. 2013). Effective implementation of VMI policy helps to schedule deliveries, inventory, order and production by integrating the information throughout the chain. However, many researchers have been using classical economic order quantity considering its simplicity and use in VMI implementation, which later on extended to match the real-time business challenges (Marque et al. 2010). However, there are several constraints imposed due to capacity in inventory systems.

VMI policy is more effective if total profit is enhanced through cost reduction. It is well known that goods deteriorate which depends on the environmental conditions. The deterioration can be slowed down through investing in preservation technology and investment in preservation affects the rate of deterioration (Yang, Dye and Ding, 2015). Paying on preservation technology (PT) to reduce deterioration rate make sense to improve customer service level, economic losses and business competitiveness. In last decade, preservation technology has received little attention. The higher deterioration rate implies higher annual relevant cost and lower demand rate (Johnny et al., 2007). Dye and Hsieh (2012) presented an extended model of (Hsu et al., 2010) by assuming that the preservation technology cost is a function of the length of replenishment cycle.

Therefore, integration of preservation technology in lot sizing decisions should be emphasized. Spurred by the profit-maximization approach, it is reasonable to study the joint optimal replenishment policy with preservation technology investment during a replenishment cycle for non-instantaneous deteriorating items for traditional system and for VMI system.

As per above arguments, proposed model consider non-instantaneous deterioration nature of the item. Preservation technology investment is provided to reduce deterioration rate. Also single vendor-single buyer vender managed inventory model is developed and total cost of the supply chain is compared with VMI system and traditional system. Notations and assumptions are provided in section 2. Model formulation with both the traditional supply chain system and VMI system is given in section 3. In section 4, numerical example is shown and sensitivity analysis is carried out followed by numerical example in section 5. At the end, section 6 concludes the study with the results and future scope.

2. Notation and assumptions

Existing model is built up under the following notations and assumptions.

2.1 Notations

Decision variables	
T	the length of replenishment cycle in traditional system
T_{VMI}	the length of replenishment cycle in VMI system
ξ	the preservation technology cost to reduce deterioration rate, $\xi \geq 0$
Parameters	
K_S	supplier's ordering cost per order
K_B	buyer's ordering cost per order
c_h	Unit holding cost of buyer per unit time
θ	constant deterioration rate
$m(\xi)$	reduced deterioration rate which is function of ξ
W	Maximum inventory level at each cycle
c_d	deterioration cost per unit
t_d	the length of deterioration free time
D_1	constant demand rate at time $t\left(0 \leq t \leq t_d\right)$
$D_1 e^{-\lambda(t-t_d)}$	demand rate at time $t\left(t_d \leq t \leq T\right)$
ZB	buyer's inventory cost in traditional system
ZB_{VMI}	buyer's inventory cost in VMI system
ZS	supplier's inventory cost in traditional system
ZS_{VMI}	supplier's inventory cost in VMI system
TC	total inventory cost before implementation of VMI system
TC_{VMI}	total inventory cost of VMI system
Q	order quantity per cycle
$I_1(t)$	the inventory level at time $t, \left(0 \leq t \leq t_d\right)$

$I_2(t)$	the inventory level at time $t,(t_d \le t \le T)$

2.2 Assumptions

1. Replenishment rate is infinite, but its order size is finite.
2. The inventory system involves single non-instantaneous deteriorating item.
3. During time period t_d, the product has no deterioration.
4. After that the on-hand inventory deteriorate with constant rate θ where $0 < \theta < 1$. Besides, t_d is considered as a given constant such that $0 \le t_d \le T$.
5. There is no provision for repair or replacement of deteriorated units.
6. The reduced deterioration rate, $m(\xi)$, is concave, continuous, strictly increasing in the level of investment in preservation technology ξ.
7. In this paper, we assume different rates of constant demand which is given by
$$D = \begin{cases} D_1 & \text{if } 0 \le t \le t_d \\ D_1 e^{-\lambda(t-t_d)} & \text{if } t_d \le t \le T \end{cases}$$
8. Shortages are not allowed.

3. Model Formulation

Present literature consider constant demand at different rate in different time interval. Original deterioration rate is θ and $m(\xi)$ is reduced deterioration rate when retailer invests ξ on the preservation technology cost in tradition inventory system and supplier invests ξ on the preservation technology cost in VMI system. Hence the resultant deterioration rate will be $(\theta - m(\xi))$. Objective of this study is to minimize the average cost by determining preservation technology strategy, replenishment cycle and order quantity in traditional system and VMI system.

As per the notations and assumptions, the behavior of the inventory system can be seen in Fig. 1 shows that Q units of the items replenish at the beginning of the inventory system. Over time interval $[0, t_d]$, the inventory level declines only due to demand at constant rate and inventory level reaches to zero due to deterioration and demand over the time interval $[t_d, T]$.

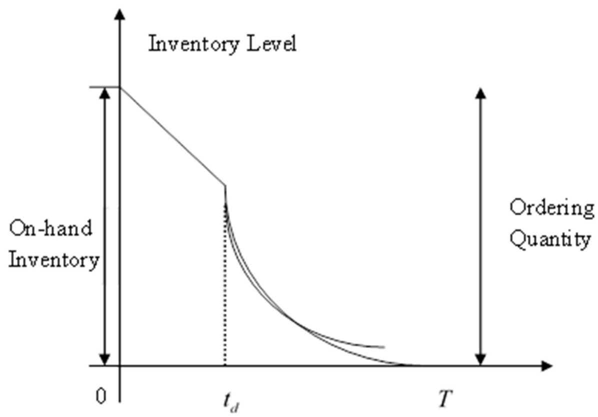

Fig. 1: The graphical representation for the inventory system

Following differential equations describe given inventory model

$$\frac{dI_1(t)}{dt} = -D_1, 0 \le t \le t_d \tag{1}$$

$$\frac{dI_2(t)}{dt} + \left(\theta - m(\xi)\right) I_2(t) = -D_1 e^{-\lambda(t-t_d)}, t_d \le t \le T \tag{2}$$

with terminal conditions $I_1(0) = W, I_2(T) = 0$.

The solution of Eqs. (1) and (2) is

$$I_1(t) = W - D_1 t, \quad 0 \le t \le t_d \tag{3}$$

$$I_2(t) = \frac{D_1 e^{\lambda t_d}}{\theta - m(\xi) - \lambda}\left(e^{-\lambda T + (\theta - m(\xi))(T-t)} - e^{-\lambda t}\right), \quad t_d \le t \le T \tag{4}$$

Since $I_1(t) = I_2(t)$ at $t = t_d$ it follows from Eq. (3) and Eq. (4)

$$W - D_1 t_d = \frac{D_1 e^{\lambda t_d}}{\theta - m(\xi) - \lambda}\left(e^{-\lambda T + (\theta - m(\xi))(T-t_d)} - e^{-\lambda t_d}\right)$$

Hence maximum inventory level W, is

$$W = D_1 t_d + \frac{D_1 e^{\lambda t_d}}{\theta - m(\xi) - \lambda}\left(e^{-\lambda T + (\theta - m(\xi))(T-t_d)} - e^{-\lambda t_d}\right) \tag{5}$$

Using Eq. (5) in Eq. (3), we get

$$I_1(t) = D_1(t_d - t) + \frac{D_1 e^{\lambda t_d}}{\theta - m(\xi) - \lambda}\left(e^{-\lambda T + (\theta - m(\xi))(T-t_d)} - e^{-\lambda t_d}\right) \tag{6}$$

Ordered quantity Q is equals to maximum inventory level W. Therefore

$$Q = D_1 t_d + \frac{D_1 e^{\lambda t_d}}{\theta - m(\xi) - \lambda}\left(e^{-\lambda T + (\theta - m(\xi))(T-t_d)} - e^{-\lambda t_d}\right) \tag{7}$$

The elements comprising the cost function per cycle are listed below:

1. Product holding cost:

$$HC = h\left[\int_0^{t_d} I_1(t)dt + \int_{t_d}^{T} I_2(t)dt\right] \tag{8}$$

2. Product deterioration cost:

$DC = c_d \times$ the amount of deteriorated units during $[t_d, T]$

$$= c_d\left(I_2(t_d) - \int_{t_d}^{T} D_1 e^{-\lambda(t-t_d)}\,dt\right) \tag{9}$$

3. Buyer's ordering cost: K_B

4. Supplier's ordering cost: K_S

5. Preservation technology cost ξ

3.1 Traditional supply chain system

Before implementation of VMI, the total cost of the buyer and the total cost of the supplier in tradition inventory model are respectively given by Eqs. (10) and (11).

$$ZB(\xi,T) = \frac{1}{T}(K_B + HC + DC + \alpha\xi)$$

$$= \frac{1}{T}\left\{\begin{array}{l} \alpha\xi + K_B + K_S + \dfrac{D_1(c_h t_d + c_d)}{\theta - m(\xi) - \lambda}\left(e^{(\theta - m(\xi) - \lambda)(T - t_d)} - 1\right) + \dfrac{c_h D_1 t_d^{\,2}}{2} + \\ \left(D_1 c_d + \dfrac{D_1 c_h}{\theta - m(\xi) - \lambda}\right)\left(\dfrac{e^{-\lambda(T-t_d)} - 1}{\lambda}\right) + \dfrac{D_1 c_h e^{-\lambda(T-t_d)}\left(e^{(\theta - m(\xi))(T-t_d)} - 1\right)}{(\theta - m(\xi) - \lambda)(\theta - m(\xi))} \end{array}\right\} \tag{10}$$

$$ZS(\xi,T) = \frac{K_S + (1-\alpha)\xi}{T} \tag{11}$$

Total average cost of the chain before VMI is denoted by $TC(\xi,T)$, is given by

$$TC(\xi,T) = ZB(\xi,T) + ZS(\xi,T)$$

For the buyer, the optimization problem in traditional supply chain system is given by

$$\begin{array}{ll} \underset{\xi,T}{Minimize} & ZB(\xi,T) \\ \text{subject to} & 0 \le t_d \le T \end{array} \tag{12}$$

To find optimal value of $(\xi,T) = (\xi^*,T^*)$, first we keep ξ fix, then total cost function of buyer (*ZB*) is only function of cycle length T. Once we find optimal value of $T = T^*$, substituting T^* in Eq. (12), we find optimal value of ξ^*.

To find optimal value of *T*, we proceed as below.

Differentiating Eq. (12) w.r.t. T, keeping ξ fixed and using Taylor series expansion, one have

$$\frac{dZB(T)}{dT}=\frac{F(T)}{T}\text{ where}$$

$$F(T)=\frac{c_h D_1 t_d^{\,2}}{2}+TD_1\left(c_h t_d+c_d\right)\left(\theta-m(\xi)-\lambda\right)\left(T-t_d\right)+TD_1\left(T-t_d\right)\left(c_h+c_d\lambda\right)$$
$$-\alpha\xi-K_B-K_S+\frac{D_1 c_h\lambda\left(t_1-t_d\right)^2\left(\lambda T+1\right)}{\theta-m(\xi)-\lambda}$$

Differentiating $F(T)$ w.r.t. T, one have procedure

$$\frac{dF(T)}{dT}=D_1\left(c_h t_d+c_d\right)\left(\theta-m(\xi)-\lambda\right)\left(2T-t_d\right)+D_1\left(T-t_d\right)\left(c_h+c_d\lambda\right)+TD_1\left(c_h+c_d\lambda\right)$$
$$+\frac{D_1 c_h\lambda^2\left(T-t_d\right)^2}{\theta-m(\xi)-\lambda}+\frac{2D_1 c_h\lambda\left(T-t_d\right)\left(\lambda T+1\right)}{\theta-m(\xi)-\lambda} \qquad (13)$$

Lemma: The buyer's cost function $ZB(T)$ is convex in T and attains its global minimum at point $T=T^*$ if $\theta-m(\xi)-\lambda>0$.

Proof: If $\theta-m(\xi)-\lambda>0$ then form Eq. (13), $\frac{dF(T)}{dT}>0$. So, $F(T)$ is increasing function of T.

Additionally $F\left(t_d\right)=\frac{1}{2}D_1 c_h t_d^{\,2}-\xi-K_B-K_S$ and $\lim_{T\to\infty}F(T)=\infty$

Now, the optimal value of cycle length T depends on $F\left(t_d\right)$.

Sub-case 1: If $F\left(t_d\right)<0$, using Intermediate value theorem, as $F(T)$ is increasing function of $T\in\left[t_d,\infty\right)$, there exists unique T_1 such that $F\left(T_1\right)=0$.

In other words, T_1 is a unique solution of $dZB(T)/dT=0$.

Sub-case 2: If $F\left(t_d\right)\geq 0$. Since $F(T)$ is increasing function of $T\in\left[t_d,\infty\right)$, $dZB(T)/dT>0,\forall T\in\left[t_d,\infty\right)$. Hence, optimal value occurs at point $T=t_d$.

On summarizing above arguments,

we have $T^*=\begin{cases}T_1, & \text{if } F\left(t_d\right)<0\\ t_d, & \text{if } F\left(t_d\right)\geq 0\end{cases}$

$$\therefore T^*=\begin{cases}T_1, & \text{if } D_1 c_h t_d^{\,2}<2\xi+2K_B+2K_S\\ t_d, & \text{if } D_1 c_h t_d^{\,2}\geq 2\xi+2K_B+2K_S\end{cases}$$

Moreover,

$$\left[\frac{d^2ZB(T)}{dT^2}\right]_{T=T^*} = D_1\left(c_h t_d + c_d\right)\left(\theta - m(\xi) - \lambda\right)\left(2T - t_d\right) + D_1\left(T - t_d\right)\left(c_h + c_d\lambda\right) + TD_1\left(c_h + c_d\lambda\right)$$
$$+\frac{D_1 c_h \lambda^2 \left(T - t_d\right)^2}{\theta - m(\xi) - \lambda} + \frac{2D_1 c_h \lambda\left(T - t_d\right)\left(\lambda T + 1\right)}{\theta - m(\xi) - \lambda}$$
$$> 0$$

This completes the proof.

Now we find optimal value of $\xi, \xi = \xi^*$. To find ξ^*, substitute optimal value of T^* in Eq. (12). Then total cost function of buyer is only function of ξ which is given by

$$ZB(\xi) = \frac{1}{T^*}\left\{\begin{array}{l} \alpha\xi + K_B + K_S + \dfrac{D_1\left(c_h t_d + c_d\right)}{\theta - m(\xi) - \lambda}\left(e^{(\theta - m(\xi) - \lambda)\left(T^* - t_d\right)} - 1\right) + \dfrac{c_h D_1 t_d^{\,2}}{2} + \\ \left(D_1 c_d + \dfrac{D_1 c_h}{\theta - m(\xi) - \lambda}\right)\left(\dfrac{e^{-\lambda\left(T^* - t_d\right)} - 1}{\lambda}\right) + \dfrac{D_1 c_h e^{-\lambda\left(T^* - t_d\right)}\left(e^{(\theta - m(\xi))\left(T^* - t_d\right)} - 1\right)}{\left(\theta - m(\xi) - \lambda\right)\left(\theta - m(\xi)\right)} \end{array}\right\}$$

Using Taylor series expansion,

$$ZB(\xi) = \frac{1}{T^*}\left\{\begin{array}{l} \alpha\xi + K_B + K_S + D_1\left(c_h t_d + c_d\right)\left(T^* - t_d\right) + \dfrac{c_h D_1 t_d^{\,2}}{2} + \\ \left(D_1 c_d + \dfrac{D_1 c_h}{\theta - m(\xi) - \lambda}\right)\left(t_d - T^*\right) + \dfrac{D_1 c_h\left(1 - \lambda\left(T^* - t_d\right)\right)\left(T^* - t_d\right)}{\left(\theta - m(\xi) - \lambda\right)} \end{array}\right\}$$

$$ZB(\xi) = \frac{1}{T^*}\left\{\alpha\xi + K_B + K_S + D_1 c_h t_d\left(T^* - t_d\right) + \frac{c_h D_1 t_d^{\,2}}{2} - \frac{D_1 c_h \lambda\left(T^* - t_d\right)^2}{\left(\theta - m(\xi) - \lambda\right)}\right\} \tag{14}$$

Differentiating Eq. (14) w.r.t. ξ , one get

$$\frac{dTC(\xi)}{d\xi} = \frac{\alpha}{T} - \frac{D_1 c_h \lambda\left(T - t_d\right)^2}{T\left(\theta - m(\xi) - \lambda\right)^2}\frac{dm(\xi)}{d\xi} \tag{15}$$

Differentiating Eq. (15) w.r.t. ξ, one get

$$\frac{d^2TC(\xi)}{d\xi^2} = -\frac{2D_1 c_h \lambda\left(T^* - t_d\right)^2}{T^*\left(\theta - m(\xi) - \lambda\right)^3}\left(\frac{dm(\xi)}{d\xi}\right)^2 - \frac{D_1 c_h \lambda\left(T^* - t_d\right)^2}{T^*\left(\theta - m(\xi) - \lambda\right)^2}\left(\frac{d^2m(\xi)}{d\xi^2}\right) \tag{16}$$

> 0 as $m''(\xi) < 0$.

Hence total cost function is convex function of ξ for given value of T which results in a minimum value.

3.2 Vender managed inventory supply chain system

In VMI system, vendor takes all decision regarding status of replenishment, buyer inform him about status of replenishment and buyer's cost is paid by supplier. So, buyer's inventory cost in VMI system is,

$$ZB_{VMI}\left(\xi_{VMI}, T_{VMI}\right) = 0 \tag{17}$$

and

$$ZS_{VMI}\left(\xi_{VMI}, T_{VMI}\right) = \frac{K_B + K_S + HC + DC + \xi}{T_{VMI}} \tag{18}$$

Hence total average cost in vendor managed inventory system is

$$TC_{VMI}\left(\xi_{VMI}, T_{VMI}\right) = ZB_{VMI}\left(\xi_{VMI}, T_{VMI}\right) + ZS_{VMI}\left(\xi_{VMI}, T_{VMI}\right)$$

$$TC\left(\xi_{VMI}, T_{VMI}\right) = \frac{1}{T_{VMI}}\left\{\begin{array}{l} \xi_{VMI} + K_B + K_S + \dfrac{D_1\left(c_h t_d + c_d\right)}{\theta - m\left(\xi_{VMI}\right) - \lambda}\left(e^{\left(\theta - m\left(\xi_{VMI}\right) - \lambda\right)\left(T_{VMI} - t_d\right)} - 1\right) + \dfrac{c_h D_1 t_d^{\,2}}{2} + \\ \left(D_1 c_d + \dfrac{D_1 c_h}{\theta - m\left(\xi_{VMI}\right) - \lambda}\right)\left(\dfrac{e^{-\lambda\left(T_{VMI} - t_d\right)} - 1}{\lambda}\right) + \dfrac{D_1 c_h e^{-\lambda\left(T_{VMI} - t_d\right)}\left(e^{\left(\theta - m\left(\xi_{VMI}\right)\right)\left(T_{VMI} - t_d\right)} - 1\right)}{\left(\theta - m\left(\xi_{VMI}\right) - \lambda\right)\left(\theta - m\left(\xi_{VMI}\right)\right)} \end{array}\right\} \tag{19}$$

The optimization problem in vendor managed supply chain system is given by

$$\underset{\xi, T}{Minimize} \quad TC\left(\xi, T\right)$$

$$\text{subject to} \quad 0 \le t_d \le T$$

Here optimal point $\left(\xi_{VMI}^{\;*}, T_{VMI}^{\;*}\right)$ can be obtained by same argument as traditional system.

4. Numerical example

To illustrate the above theoretical result, we solve following numerical example by assuming $m\left(\xi\right) = k\left(1 - e^{-a\xi}\right), a \ge 0$ where a is a coefficient representing the percentage increase in $m\left(\xi\right)$ per dollar increase in ξ, which means $m\left(\xi\right)$ is an increasing function bounded above by k (Hsu et al., 2010) . Using proper software, one can solve following numerical examples.

Example 1: Parameters value are given as below:
$K_S = \$200$/order, $K_B = \$50$/order, $\theta = 0.98$, $c_d = \$10$/unit, $D_1 = 1000$ unit/year, $a = 0.02$, $c_h = \$3.5$/unit/year, $t_d = \dfrac{20}{365}$, $\lambda = 0.01$ and $k = 0.02$. The result of solution procedure keeping $\xi = 50$ fix is as under:

Table 1: Results of computation

Q^*	$Q_{VMI}^{\;*}$	T^*	$T_{VMI}^{\;*}$	TC^*	$TC_{VMI}^{\;*}$

120	220	0.1176	0.2084	2926.93	2400.31

From Table 1 we can say that vendor managed inventory system is more beneficial compare to individual system as cost with VMI system is less than to cost with traditional system.

Example 2: This Example shows effect of changes in preservation technology cost ξ . We consider same data of Example 1. The value of ξ is chosen form the interval [30,70] with step size 10. The impact of preservation technology cost is shown in Table 2.

Table 2: Effect of changes in ξ on decision variables

ξ	Q^*	$Q_{VMI}{}^*$	T^*	$T_{VMI}{}^*$	TC^*	$TC_{VMI}{}^*$
30	111	213	0.1100	0.2018	2878.22	2305.08
40	116	217	0.1139	0.2051	2901.33	2352.99
50	120	220	0.1176	0.2084	2926.93	2400.32
60	123	224	0.1212	0.2116	2954.52	2447.06
70	127	228	0.1246	0.2147	2983.69	2493.24

Table 2 shows that increasing value of ξ, increases all decision variables: order quantity, cycle length and total cost of supply chain in both traditional and VMI model.

5. Sensitivity analysis

To check liability of the model sensitivity analysis done and given in Table 3 in which one parameter is changed at the rate of -40%, -20%, 0%, 20% and 40% from the original value given as in Example 1 and other parameters are fix.

Table 3: **Sensitivity analysis**

Parameter	% changes	Q^*	$Q_{VMI}{}^*$	T^*	$T_{VMI}{}^*$	TC^*	$TC_{VMI}{}^*$
K_B	-40	106	213	0.1048	0.2021	2974.91	2302.87
	-20	113	217	0.1114	0.2053	2943.37	2351.97
	0	120	220	0.1176	0.2084	2926.93	2400.32
	+20	126	224	0.1234	0.2115	2921.42	2447.95
	+40	132	227	0.1290	0.2145	2920.03	2494.90
θ	-40	138	260	0.1362	0.2489	2586.58	2110.73
	-20	128	237	0.1256	0.2258	2769.52	2265.26
	0	120	220	0.1176	0.2084	2926.93	2400.32
	+20	113	207	0.1113	0.1946	3064.67	2520.32
	+40	108	195	0.1063	0.1835	3186.72	2628.28
α	-40	112	220	0.1101	0.2084	3057.28	2400.32
	-20	116	220	0.1139	0.2084	2987.93	2400.32
	0	120	220	0.1176	0.2084	2926.93	2400.32
	+20	123	220	0.1211	0.2084	2872.98	2400.32
	+40	127	220	0.1246	0.2084	2825.02	2400.32
c_d	-40	138	260	0.1345	0.2423	2612.99	2139.24
	-20	127	238	0.1249	0.2233	2781.13	2278.22
	0	120	220	0.1176	0.2084	2926.93	2400.32
	+20	113	206	0.1117	0.1964	3055.35	2509.29

	+40	108	195	0.1070	0.1865	3169.82	2607.71
D_1	-40	90	170	0.1458	0.2620	2385.77	1949.92
	-20	105	197	0.1290	0.2303	2680.40	2193.75
	0	120	220	0.1176	0.2084	2926.93	2400.32
	+20	133	242	0.1093	0.1921	3139.62	2580.79
	+40	146	262	0.1029	0.1794	3326.94	2741.77
a	-40	119	220	0.1175	0.2081	2929.64	2402.66
	-20	119	220	0.1175	0.2083	2928.15	2401.37
	0	120	220	0.1176	0.2084	2926.93	2400.32
	+20	120	220	0.1176	0.2085	2925.93	2399.45
	+40	120	220	0.1177	0.2086	2925.12	2398.75
c_h	-40	127	234	0.1243	0.2201	2738.71	2241.54
	-20	123	227	0.1208	0.2140	2834.21	2322.10
	0	120	220	0.1176	0.2084	2926.93	2400.32
	+20	116	214	0.1146	0.2032	3017.10	2476.39
	+40	113	209	0.1118	0.1984	3104.91	2550.49
t_d	-40	114	219	0.1111	0.2040	3173.98	2583.50
	-20	116	219	0.1140	0.2060	3049.65	2488.93
	0	120	220	0.1176	0.2084	2926.93	2400.32
	+20	123	222	0.1218	0.2111	2807.38	2317.48
	+40	128	224	0.1265	0.2142	2692.33	2240.20
λ	-40	120	220	0.1176	0.2084	2926.93	2400.32
	-20	120	220	0.1176	0.2084	2926.93	2400.32
	0	120	220	0.1176	0.2084	2926.93	2400.32
	+20	120	220	0.1176	0.2084	2926.93	2400.32
	+40	120	220	0.1176	0.2084	2926.93	2400.32
k	-40	119	220	0.1174	0.2080	2930.71	2403.59
	-20	119	220	0.1175	0.2082	2928.82	2401.95
	0	120	220	0.1176	0.2084	2926.93	2400.32
	+20	120	220	0.1177	0.2086	2925.04	2398.68
	+40	120	221	0.1178	0.2088	2923.14	2397.03

- The analysis shows that optimal value of order quantity Q^* in traditional supply chain system increases with increases in K_B, α, D_1, a, t_d and k and decreases with increases in θ, c_d and c_h. However Q^* remains constant with changes in λ. In vendor managed inventory system, $Q_{VMI}{}^*$ increases with increases in K_B, D_1, t_d and k and decreases in θ, c_d and c_h. However it remains unchanged with respect to α, a and λ which shows that α, a and λ are highly insensitive parameters. Analysis revels that order quantity increases in vendor managed inventory system compare to traditional system.
- Cycle length T^* and $T_{VMI}{}^*$ increases with decreases in θ, c_d, D_1 and c_h and increases in K_B, a, t_d and k. T^* and $T_{VMI}{}^*$ are remains as it is with changes in λ. With increases in α , Cycle length increases in traditional supply chain system but does not change in VMI system. However cycle length

increases in vendor managed inventory system compare to the system developed by individual effort of vendor and buyer.

- Total cost TC^* and $TC_{VMI}{}^*$ increases with increases in θ, c_d, D_1 and c_h and decreases with increases in a, t_d and k. However total cost remains unchanged in both the models, traditional inventory model and VMI model with changes in λ. Hence λ is highly insensitive parameter. Total cost in traditional model increases with decreases in α and remains same in VMI system. Analysis reveals that demand parameter D_1 is highly sensitive in vendor managed inventory system. However, vendor managed inventory supply chain system reduces total cost in compare of the system developed by individual effort of vendor and buyer.

6. Conclusion and future scope:

Present study considered investment in preservation technology and replenishment schedule as decision variables and formulated total cost with traditional supply chain system and vendor managed inventory supply chain system for non-instantaneous deteriorating items where original deterioration rate is constant. Demand is constant before deterioration starts and after that it decreases exponentially. Total cost and decision variables are compared in both models, traditional model and VMI model. For any given preservation technology cost, we show existence of optimal replenishment schedule theoretically and numerically. Then for the given replenishment schedule, we proved total cost function is convex function of preservation technology cost. We gave numerical examples and sensitivity analysis to validate the model. Results show that VMI supply chain system is beneficial compare to traditional supply chain system.

We can extend the model by assuming time dependent variable deterioration rate, demand is sock sensitive.

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