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Application of Queueing Theory in Decision Making in Presence of Uncertain Environment

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Abstract

In the present uncertain business environment, behavior of customers is one of the most uncertain phenomena. Customer impatience is also one of the uncertain phenomena which is a strong threat for business in present era of business. Impatient customers results in loss of customers for business industries. In this paper, we construct a stochastic queueing model with multi-server and finite capacity. Reverse balking is the new concept introduced in the stochastic queueing models. Reneging is the one of the most known phenomenon in queueing theory. In this paper, the concept of reverse balking and impatient behavior of the customers is incorporated in the constructed queueing model. The steady state solution of the model has been developed by using iterative procedure and some important performance measures of the queuing system like expected size of the system, the average rate of reneging, average waiting time of the customer in queue etc. are derived for the finite capacity with multi-server queuing model.

Keywords: Reverse balking, customer impatience, stochastic modeling, queueing theory, steady state solution.

Mathematics Subject Classification (MSC) 2010: 54D45, 60Gxx, 90Bxx, 60Jxx

1. Introduction

Queueing models are effectively used in the design and analysis of service system, telecommunication system, traffic system and many more. Queueing theory is a Mathematical study of waiting lines or queues. The formation of a queue is a common phenomenon which occurs whenever the current demand for service exceeds the current capacity to provide that service. Queues may be seen at railway station, cinema ticket windows, billing counters of super market etc. The main objective of queueing theory is to minimize the congestion and provide best level of service at minimum possible cost. The waiting lines arise from congestion which occurs from time to time as a result of irregularities in arrivals or in the length of time needed to service customers. Assuming that the irregularities mentioned above follow some probabilistic laws, the queueing theory attempts to study statistical fluctuations in these irregularities. It also helps to determine the probability of number of customers in the system, the average system size, the average queue length, average waiting time of customers in the system, the average system size, average server utilization etc.

Now a day's managing business has become a challenging task due to highly uncertain economic environment in industries. One of the most uncertain aspects of business is customer's behavior. Customers have become more selective in present era. A customer may get impatient due to higher level of expectations, delay in service, lack of facility otherwise he decides to leave the service facility before completion of service. This type of behavior leads to the loss in profit and goodwill of the company and it becomes the most important threat to any business industries. But, when it comes to sensitive businesses like investment, selecting a restaurant for dinner, selecting a service center, choosing a saloon etc. more number of customers with a particular firm becomes the attracting factor for other customers. Level of impatience of customers depends upon the amount of trust they show with particular company. Customers are willing to spend more time with high level of patience with the firms having a large consumer's base. For instance, if someone is planning to dine out, he is willing to wait for much longer in order to get access to a well-known restaurant. It is also obvious that well-known brands have a large customers' base. Hence, a large customers' base also works as a trust factor for a customer and the patience level of the customer is high in such type of cases. This behaviour is referred to as reverse reneging, according to which, higher system size results in high patience and vice-versa. By keeping this in mind, researchers across the world study various stochastic queuing models with reneging. In this model, reneging is a function of system size. When there is more number of customers in the system and vice versa, which is reneging in reverse sense called as Reverse Reneging. Barrer [1] obtained the steady - state solution of a single channel queuing model having poisson input, exponential holding time, random selection where impatient customers leave the service facility after a wait of certain time. Hunt [2] studied the problems of serial queues in the steady state with Poisson assumptions. In these studies it is assumed that the unit must go through each service channel without leaving the system.

The busy period for the Poisson queue is studied by Takcacs [3]. Some queuing problems with balking and reneging I was studied by Ancker and Gafarian [4]. Ancker and Gafarian [5] obtain results for a pure balking system by setting the reneging parameter equal to zero. The premier work on customer impatience in queuing theory appears in Haight [6]. He investigated M/M/1 queue with balking in which there is a greatest queue length at which the arrival will not balk. Haight [7] studies a queue with reneging in which he studies the problem like how to make rational decision while waiting in the queue, the probable effect of this decision etc. Kumar [8] discussed the busy period analysis of an M/M/1 queue with balking. Armony et al [9] study sensitivity of the optimal capacity to customer impatience. They observe that the prevention of reneging during service can substantially reduce the total cost of lost sales and capacity. Choudhury and Medhi [10] presented some aspects of balking and reneging in finite buffer queue. Wang et al. [11] performed reliability analysis using Laplace transform techniques. Recently, Rakesh and Sumeet [12] studied a queuing model with retention of reneged customers and balking. Kumar & Sharma [13] developed queueing model with reneging, balking and retention of reneged cstomers. They also developed some performance measures and analyzed the results numerically. Awasthi [14] analyzed performance of M/M/1/K finite capacity model with reverse balking and reverse reneging. He developed steady solution of the model and some performance measures of the model also described.

In this paper, we present steady-state analysis of the stochastic model and derive some important measures of performance for the queueing model. Rest of the paper is arranged as follows: In section 2, the assumptions of the queueing model are described. In section 3, Mathematical Analysis of the queueing model has been done. Steady state solution of the model has been developed in section 4. In section 3, Mathematical Analysis of the queueing model has been done. In section 5, Performance measures for Queueing model have been derived. Conclusion of the research paper is described in section 6.

2. Assumptions of the Queueing model

Assumptions of the queueing model are as follows:

1) Customers arrive in the queueing system one by one according to a Poisson distribution with mean arrival rate λ . The inter-arrival times are independently, identically and exponentially distributed with parameter λ .
2) There are c servers in the service facility of the queueing system. The service times are independently, identically and exponentially distributed with parameter μ such as
3) $\mu = \begin{cases} k\mu, \; for \; k \prec c \\ c\mu, \; for \; k \geq c \end{cases}$
4) The capacity of the queueing system is K.
5) Service discipline of the queueing system is First Come, First served.
6) When the system is empty, the customer may balk with probability b_1 and may reside in the system to get service with probability b_2 .
7) When there is at least one customer in the queueing system, the customer balk with the probability $\left(1-\frac{k}{K-1}\right)$ and join the queueing system with probability $\left(\frac{k}{K-1}\right)$. Such types of behaviour of the customers are known as reverse balking.

8) The customers may get impatient after some time, say t due to certain reasons and decide to leave the queue before getting service. The reneging time (t) is independent, identically and exponentially distributed with parameter υ.

9) $P_k(t)$ be the probability that there is k customer in the system at time t.

10) $P_0(t)$ be the probability that there is no customer in the system at time t.

3. Mathematical Analysis of the queueing model

The governing differential difference equations of the system are

$$\frac{dP_0(t)}{dt} = -\lambda b_2 P_0(t) + \mu\, P_1(t); \quad for\, k = 0 \tag{1}$$

$$\frac{dP_1(t)}{dt} = \lambda b_2 P_0(t) - \left[\left(\frac{1}{K-1}\right)\lambda + \mu\right] P_1(t) + 2\mu\, P_2(t); \quad for\, k = 1 \tag{2}$$

$$\frac{dP_k(t)}{dt} = \left(\frac{k-1}{K-1}\right)\lambda\, P_{k-1}(t) - \left[\left(\frac{k}{K-1}\right)\lambda + k\mu\right] P_k(t) + [(k+1)\mu] P_{k+1}(t); \quad 2 \le k \prec c \tag{3}$$

$$\frac{dP_k(t)}{dt} = \left(\frac{k-1}{K-1}\right)\lambda\, P_{k-1}(t) - \left[\left(\frac{k}{K-1}\right)\lambda + \{c\mu + (k-c)\upsilon\}\right] P_k(t) + [c\mu + \{(k+1)-c\}\upsilon] P_{k+1}(t); \quad c \le k \prec K-1 \tag{4}$$

$$\frac{dP_K(t)}{dt} = \lambda P_{K-1}(t) - [c\mu + (K-c)\upsilon] P_K(t); \quad for\, k = K \tag{5}$$

4. Steady State Solutions of the queueing system

In the steady state position, $\lim_{k\to\infty} P_k(t) = P_k$.

Therefore, $0 = -\lambda b_2 P_0(t) + \mu\, P_1(t); \quad for\, k = 0$ (6)

$$0 = \lambda b_2 P_0(t) - \left[\left(\frac{1}{K-1}\right)\lambda + \mu\right] P_1(t) + 2\mu\, P_2(t); \quad for\, k = 1 \tag{7}$$

$$0 = \left(\frac{k-1}{K-1}\right)\lambda\, P_{k-1}(t) - \left[\left(\frac{k}{K-1}\right)\lambda + k\mu\right] P_k(t) + [(k+1)\mu] P_{k+1}(t); \quad 2 \le k \prec c \tag{8}$$

$$0 = \left(\frac{k-1}{K-1}\right)\lambda\, P_{k-1}(t) - \left[\left(\frac{k}{K-1}\right)\lambda + \{c\mu + (k-c)\upsilon\}\right] P_k(t) + [c\mu + \{(k+1)-c\}\upsilon] P_{k+1}(t); \quad c \le k \prec K-1 \tag{9}$$

$$0 = \lambda P_{K-1}(t) - [c\mu + (K-c)\upsilon] P_K(t); \quad for\, k = K \tag{10}$$

Solving equations (6)-(10), the steady state solution of the model is given by

$$P_k = \begin{cases} \left[\dfrac{(k-1)!}{(K-1)^{k-1}} \displaystyle\prod_{j=1}^{k} \frac{\lambda}{j\,\mu}\right] b_2\, P_0, \; k \prec c \\ \left[\dfrac{(k-1)!}{(K-1)^{k-1}} \displaystyle\prod_{i=c}^{k} \frac{\lambda}{[c\,\mu + (i-c)\upsilon]} \prod_{j=1}^{c-1} \frac{\lambda}{j\,\mu}\right] b_2\, P_0, \; k \ge c \\ \left[\dfrac{(K-2)!}{(K-1)^{K-1}} \displaystyle\prod_{i=c}^{k} \frac{\lambda}{[c\,\mu + (i-c)\upsilon]} \prod_{j=1}^{c-1} \frac{\lambda}{j\,\mu}\right] b_2\, P_0, \; k = K \end{cases} \tag{11}$$

Using normalization condition $\sum_{i=0}^{K} P_k = 1$

$$or\ P_0 + \sum_{k=1}^{c-1} P_k + \sum_{k=c}^{K-1} P_k + P_K = 1$$

$$Or P_0 = \frac{1}{\left[\left\{\left[\frac{(k-1)!}{(K-1)^{k-1}}\prod_{j=1}^{k}\frac{\lambda}{j\mu}\right]\right\}+\left\{\left[\frac{(k-1)!}{(K-1)^{k-1}}\prod_{i=c}^{k}\frac{\lambda}{[c\mu+(i-c)\upsilon]}\prod_{j=1}^{c-1}\frac{\lambda}{j\mu}\right]\right\}+\left\{\left[\frac{(K-2)!}{(K-1)^{K-1}}\prod_{i=c}^{k}\frac{\lambda}{[c\mu+(i-c)\upsilon]}\prod_{j=1}^{c-1}\frac{\lambda}{j\mu}\right]\right\}b_2\right]} \tag{12}$$

5. Performance Measures of the queueing system

Here we derive some important measures of performance

5.1 Average number of customers in a system (Expected Size of the System)

Expected number of customers in the system is known as system size.

$$L_s = \sum_{i=1}^{K} kP_k$$

$$= \sum_{k=1}^{c-1} k\,P_k + \sum_{k=c}^{K-1} k\,P_k + KP_K$$

$$= \sum_{k=1}^{c-1} k\left[\frac{(k-1)!}{(K-1)^{k-1}}\prod_{j=1}^{k}\frac{\lambda}{j\mu}\right]b_2\,P_0 + \sum_{k=c}^{K-1} k\left[\frac{(k-1)!}{(K-1)^{k-1}}\prod_{i=c}^{k}\frac{\lambda}{[c\mu+(i-c)\upsilon]}\prod_{j=1}^{c-1}\frac{\lambda}{j\mu}\right]b_2\,P_0$$

$$+\mathrm{K}\left[\frac{(K-2)!}{(K-1)^{K-1}}\prod_{i=c}^{k}\frac{\lambda}{[c\mu+(i-c)\upsilon]}\prod_{j=1}^{c-1}\frac{\lambda}{j\mu}\right]b_2\,P_0 \tag{13}$$

5.2 Average Rate of Reverse Balking

Average rate of reverse baking means average number of customers after getting positive feedback of the company.

$$R_b = b_1\lambda P_0 + \sum_{k=1}^{K-1}\left(1-\frac{k}{K-1}\right)\lambda\,P_k \tag{14}$$

5.3 Average rate of reneging

Average rate of reneging of the queueing system is given by

$$R_r = \sum_{k=c}^{K}(k-c)\left\{\left[\frac{(k-1)!}{(K-1)^{k-1}}\prod_{i=c}^{k}\frac{\lambda}{[c\mu+(i-c)\upsilon]}\prod_{j=1}^{c-1}\frac{\lambda}{j\mu}\right]\right\}\upsilon b_2 P_0 + (K-c)\left[\frac{(k-1)!}{(K-1)^{k-1}}\prod_{i=c}^{k}\frac{\lambda}{[c\mu+(i-c)\upsilon]}\prod_{j=1}^{c-1}\frac{\lambda}{j\mu}\right]\upsilon b_2 P_0 \tag{15}$$

5.4 Waiting time of the customer in the system

Waiting time in the system is qiven by

$$W = W_q + \frac{1}{\mu}$$

$$= \frac{L_q}{\lambda} + \frac{1}{\mu}$$

$$= \frac{\sum_{k=c}^{K}(k-c)\left\{\left[\frac{(k-1)!}{(K-1)^{k-1}}\prod_{i=c}^{k}\frac{\lambda}{[c\mu+(i-c)\upsilon]}\prod_{j=1}^{c-1}\frac{\lambda}{j\mu}\right]b_2\right\}P_0}{\lambda} + \frac{1}{\mu} \tag{16}$$

5.5 Average waiting time of the customer in the queue

Average waiting time in the queue is given by

$$W_q = \frac{L_q}{\lambda} = \frac{1}{\lambda}\left[\sum_{k=c}^{K}(k-c)\left\{\left[\frac{(k-1)!}{(K-1)^{k-1}}\prod_{i=c}^{k}\frac{\lambda}{[c\,\mu+(i-c)\upsilon]}\prod_{j=1}^{c-1}\frac{\lambda}{j\,\mu}\right]b_2\right\}P_0\right] \tag{17}$$

5.6 Average number of customer in the queue

In a queueing system, there will be no queue formed till the number of customers are less than or equal to the number of servers. The customer will enter in the queue when on the arrival in the system a customer will find all the servers busy.

The aversge number of customer in the queue is given by

$$L_q = \sum_{k=c}^{K}(k-c)P_k$$

$$= \sum_{k=c}^{K}(k-c)\left\{\left[\frac{(k-1)!}{(K-1)^{k-1}}\prod_{i=c}^{k}\frac{\lambda}{[c\,\mu+(i-c)\upsilon]}\prod_{j=1}^{c-1}\frac{\lambda}{j\,\mu}\right]b_2\right\}P_0 \tag{18}$$

5.7 Ideal time of server

The probability that all the servers are idle is

$$P_0 = \frac{1}{\left[\left\{\left[\frac{(k-1)!}{(K-1)^{k-1}}\prod_{j=1}^{k}\frac{\lambda}{j\mu}\right]\right\}+\left\{\left[\frac{(k-1)!}{(K-1)^{k-1}}\prod_{i=c}^{k}\frac{\lambda}{[c\mu+(i-c)\upsilon]}\prod_{j=1}^{c-1}\frac{\lambda}{j\mu}\right]\right\}+\left\{\left[\frac{(K-2)!}{(K-1)^{K-1}}\prod_{i=c}^{k}\frac{\lambda}{[c\mu+(i-c)\upsilon]}\prod_{j=1}^{c-1}\frac{\lambda}{j\mu}\right]\right\}b_2\right]} \tag{19}$$

6. Conclusion

In this paper, a multiserver Markovian queueing model with reverse balking and reneging of customer's is developed. Queueing model is developed for finite capacity with multiple servers. Steady state solution of the model is derived by using iterative procedure. Some important performance measures have been derived for the stochastic model to analyze the effect of different parameters on the model developed. This analysis can be used in industries as a groth stategies. In future cost analysis of the model can be presented with optimization. This model is limited to finite capacity. The infinite capacity model can be developed.

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