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Heat and Mass Transfer Effects on Unsteady Free Convective MHD Flow of a Micro Polar Fluid between Two Vertical Walls

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Abstract

The present paper deals with heat and mass transfer effects on unsteady free convective MHD flow of incompressible micro polar fluid and motion takes place due to the buoyancy force between two vertical walls. Using similarity transformation governing dimensionless momentum, angular momentum, energy and concentration equations are converted to the system of linear partial differential equations with impose initial boundary conditions. For validation of numerical solutions obtained from Matlab software, analytical expression of these problems for steady state case has been obtained and compare through tabular form. The effects of different physical parameter on velocity, micro-rotation, temperature and concentration profiles are discussed through several figures. We found that magnetic field M, vortex viscosity R and its steady state time has retarding effect on velocity and angular velocity of micro polar fluid.

Keywords: Micro polar fluid; unsteady flow; Magnetic field.

AMS Mathematics Subject Classification: 76W05, 76W99

NOMENCLATURE

y' Co-ordinate perpendicular of the walls

L Distance between two vertical walls

B_0 Uniform magnetic field

t' Time

t Time in non-dimensional form

β'_T Volumetric coefficient of thermal expansion

β'_C Volumetric coefficient of mass expansion

y Dimensionless co-ordinate perpendicular to the walls

k Vortex viscosity

R Vortex viscosity parameter

w' angular velocity
w Dimensionless angular velocity
T_c' Temperature of the wall at $y' = L$
T_h' Temperature of the wall at $y' = 0$
T_m' Initial temperature of the fluid
T' Fluid temperature
θ Dimensionless fluid temperature
C_c' Concentration of the wall at $y' = L$
C_h' Concentration of the wall at $y' = 0$
C_m' Initial Concentration of the fluid
C' Concentration
C Dimensionless Concentration
j Micro-inertia density
v Kinematic viscosity of the fluid
σ Electrical conductivity
b Material parameter
m Temperature ratio
m_1 Concentration ratio
C_p Specific heat at constant pressure
u' Velocity of fluid
u Fluid velocity in non-dimensional form
μ plastic dynamic viscosity
D Mass diffusion coefficient
Gr Thermal Grashof number
Gm Mass Grashof number
Pr Prandtl number
M Magnetic parameter
Sc Schmidt number
g Acceleration due to gravity
ρ Fluid density

1. Introduction

Micro polar fluids are fluids with microstructure. Physically, micro polar fluids may represent fluids consisting of rigid, randomly oriented particles suspended in a viscous medium, where the deformation of fluid particles is ignored. Eringen [1] introduced the micro polar fluid theory which is useful for explaining the characteristics of certain fluids such as liquid crystals, suspensions and animal blood. Chamkha et al. [2] and Abdulaziz and Hashim [3] studied micro-polar fluid in vertical channel whereas, Ravi et al. [4] considered effects of heat transfer on free convective micro polar fluid between two vertical walls. Ching-Yang [5] considered fully develop free convective micro polar fluid with heat and mass transfer.

MHD is the combination of hydrodynamics and electromagnetic theory. Research works in the magneto hydrodynamics have been advanced significantly during the last few years in natural sciences and engineering disciplines. Hartmann [6] first introduced liquid metal duct flows under the influence of a strong external magnetic field. Joaquín et al. [7] studied Unsteady MHD free convection of a micro polar fluid between two parallel porous vertical walls whereas, Kataria et al. [8] deal with unsteady free convective MHD micropolar fluid between two vertical walls. Ellahi et al. [9] discussed influence of heat and mass transfer on micro polar fluid of blood flow with permeable walls. Ezzat et al. [10] studied with unsteady MHD flow of natural convection heat and mass transfer through a porous medium saturated with a viscoelastic fluid. Sheikholeslami et al. [11] discussed micro polar fluid behaviors on steady MHD free convection and mass transfer flow. Loganathan et al. [12] considered nanofluid past a vertical plate in the presence of heat generation whereas, Khan et al.

[13] studied MHD flow and heat transfer of a viscous fluid in a porous medium. Abbas et al. [14] considered chemically reactive hydromagnetic flow of a second-grade fluid in a semi-porous channel. Khademinejad et al. [15] obtained the solution of heat transfer in a liquid film over an unsteady stretching sheet using homotopy analysis method. Borrelli et al. [16] studied MHD of a micro polar fluid in a vertical channel. The radiations due to heat transfer effects on different flows are very important in space technology and high temperature processes. Thermal radiation parameter effects may play an important role in controlling heat transfer in polymer processing industry. Narayana et al. [17] and Olajuwon et al. [18] considered effects of Hall current on MHD micro polar fluid. Prakash et al. [19] discussed radiation effects on transient MHD flow of micropolar fluid. Kataria and Patel [20-21] studied effect of radiation and chemical reaction on MHD Casson fluid flow with ramped wall temperature, whereas Kataria and Patel [22-23] considered MHD Second grade fluid flow with ramped wall temperature in porous medium. Kataria and Mittal [24-25] studied with unsteady free convective MHD nano fluid flow through porous medium. Sheikholeslami et al. [26] defined the three dimensional MHD nanofluid flow with thermal radiation. Das and Sarkar [27] considered effect of melting on an MHD micropolar fluid flow toward a shrinking sheet with thermal radiation.

. In most of the research works, case of asymmetric or symmetric thermal condition are studied in absence of magnetic field and concentration. In this paper, we have analyzed effect of magnetic field, thermal radiation and concentration on unsteady natural convective flow of micro polar fluid between infinitely long vertical walls for asymmetric/symmetric distribution of temperature and concentration.

2. Basic equations and description of the problem

Consider the effects of heat and mass transfer on unsteady free-convective flow of micro-polar fluid between two insulated vertical walls separated by L distance with magnetic field. As shown in Fig. 1, the coordinate system is chosen, x' measures the distance along the walls and y' measures the distance normal to it. A constant uniformly distributed transverse magnetic field of strength B_0 is applied in the y'-direction. Initially, when $t' = 0,$ the temperature and concentration of walls of the fluid are says $T_f{}'$ & $C_f{}'$ respectively. When time $t' > 0,$ the temperature and concentration of the walls at $y' = 0$ and $y' = L$ is instantaneously raised and lowered to $T_h{}'$ & $C_h{}'$ and $T_c{}'$ & $C_c{}'$ respectively such that $T_h{}' > T_c{}'$ *and* $C_h{}' > C_c{}'$ which is there after maintained constant.

Under above assumptions and taking into account the Boussinesq and boundary layer approximations, momentum, angular momentum, energy and concentration equations of micro polar fluid can be expressed as follows:

$$\rho\frac{\partial u'}{\partial t'} = (\mu + k)\frac{\partial^2 u'}{\partial y'^2} + k\frac{\partial w'}{\partial y'} + \rho g\beta_T(T' - T_m{}') + \rho g\beta_C\left(C' - C_{m_1}{}'\right) - B_0{}^2\sigma u' \tag{1}$$

$$\rho j\frac{\partial w'}{\partial t'} = \left(\mu + \frac{k}{2}\right)j\frac{\partial^2 w'}{\partial y'^2} - k\left(2w' + \frac{\partial u'}{\partial y'}\right) \tag{2}$$

$$\frac{\partial T'}{\partial t'} = \alpha \frac{\partial^2 T'}{\partial y'^2} \qquad (3)$$

$$\frac{\partial C'}{\partial t'} = D \frac{\partial^2 C'}{\partial y'^2} \qquad (4)$$

Initial & boundary conditions are

$$t \le 0 \quad u' = w' = 0, \quad T' = T_f', \quad C' = C_f', \quad 0 \le y' \le 1$$

$$t > 0 \quad u' = w' = 0, \quad T' = T_h', \quad C' = C_h', \quad y' = 0$$

$$u' = w' = 0, \quad T' = T_c', \quad C' = C_c', \quad y' = L \qquad (5)$$

Introducing the following similarity transformations in Equations 1-5,

$$y = \frac{y'}{L}, \qquad t = \frac{vt'}{L^2} \qquad u = \frac{vu'}{L^2}$$

$$w = \frac{v\,w'}{L} \qquad \theta = \frac{T'-T_m'}{T_h'-T_m'} \qquad C = \frac{C'-C_m'}{C_h'-C_{m_1}'}$$

$$m' = \frac{T_c'-T_m'}{T_h'-T_m'} \qquad m_1' = \frac{C_c'-C_m'}{C_h'-C_m'} \qquad Gr = \frac{g\,\beta_T(T_h'-T_m')}{v^2}$$

$$Gm = \frac{g\,\beta_C(C_h'-C_{m_1}')}{v^2} \qquad M = \frac{B_0{}^2\sigma L^2}{\mu} \qquad \mu = v\rho$$

$$R = \frac{k}{\mu} \qquad b = \frac{L^2}{j} \qquad \alpha = \frac{k}{\rho c_p}$$

$$Pr = \frac{v}{\alpha} \qquad Sc = \frac{v}{D}$$

We get the following linear system of differential equations.

$$\frac{\partial u}{\partial t} = (1+R)\frac{\partial^2 u}{\partial y^2} + Gr\,\theta + Gm\,C + R\frac{\partial w}{\partial y} - M^2 u \qquad (6)$$

$$\frac{\partial w}{\partial t} = (1+0.5\,R)\frac{\partial^2 w}{\partial y^2} - Rb\left(\frac{\partial u}{\partial y} + 2w\right) \qquad (7)$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{Pr}\frac{\partial^2 \theta}{\partial y^2} \qquad (8)$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc}\frac{\partial^2 C}{\partial y^2} \qquad (9)$$

The corresponding initial boundary conditions (5) to the considered model reduces as follows

$$t \le 0 \quad u = w = \theta = C = 0 \quad 0 \le y \le 1;$$

$$t > 0 \quad u = w = 0, \quad \theta = C = 1; \quad y = 0$$

$$u = w = 0, \quad \theta = m, \quad C = m_1; \quad y = 1 \qquad (10)$$

The physical quantities used in the above equations are defined in the nomenclature.

3. Numerical solution

The governing system of linear partial differential equations (6) - (9) with initial and boundary conditions equation (10) are solved numerically by using Matlab software. This attending problem requests the solution on mesh produced by spaced points from the spatial interval 31 values of y from the space interval [0, 1] and 21 values of t from the time interval [0, 2].

4. Steady state Analytic solution

In order to check the accuracy of the approximation result obtained with Matlab software, we compare the steady-state numerical results with the analytic results of the corresponding study flow. As it is steady state solution, left hand side of Equations (6) – (9) become zero then the system of partial differential equations becomes system of linear ordinary differential equations.

From Appendix we get the steady state analytic result as follows:

Case-I $a_1 \neq 0, a_2 \neq 0$

$k > 0$, $k = p^2$

(i) $p_1 > 0$ & $p_2 > 0$

$$w = c_1 e^{\sqrt{p_1}y} + c_2 e^{-\sqrt{p_1}y} + c_3 e^{-\sqrt{p_2}y} + c_4 e^{-\sqrt{p_2}y} + A \quad (11)$$

$$u = b_{10} e^{p_1 y} - b_{11} e^{-p_1 y} + c_3 b_6 e^{p_2 y} - c_4 b_6 e^{-p_2 y} + A_6 y + A_7 \quad (12)$$

(ii) $p_1 > 0$ & $p_2 < 0$

$$w = c_5 e^{\sqrt{p_1}y} + c_6 e^{-\sqrt{p_1}y} + c_7 cos\sqrt{p_2}y + c_8 sin\sqrt{p_2}y + A \quad (13)$$

$$u = b_{15} e^{p_1 y} - b_{16} e^{-p_1 y} + c_7 b_{12} sinp_2 y - c_8 b_{12} cosp_2 y + A_6 y + A_7 \quad (14)$$

(iii) $p_1 < 0$ & $p_2 > 0$

$$w = c_9 cos\sqrt{p_1}y + c_{10} sin\sqrt{p_1}y + c_{11} e^{-\sqrt{p_2}y} + c_{12} e^{-\sqrt{p_2}y} + A \quad (15)$$

$$u = b_{18} sinp_1 y + b_{19} cosp_1 y + c_{11} b_6 \, e^{p_2 y} - c_{12} b_6 \, e^{-p_2 y} + A_6 y + A_7 \quad (16)$$

(iv) $p_1 < 0$ & $p_2 < 0$

$$w = c_{13} cos\sqrt{p_1}y + c_{14} sin\sqrt{p_1}y + c_{15} cos\sqrt{p_2}y + c_{16} sin\sqrt{p_2}y + A \quad (17)$$

$$u = b_{20} \, sinp_1 y + b_{21} \, cosp_1 y + c_{15} b_{12} \sin p_2 y \; e^{p_2 y} - c_{16} b_{12} \cos p_2 y + A_6 y + A_7 \quad (18)$$

$k = 0$

(i) $a_1 > 0$

$$w = (c_{17} + c_{18} y) e^{\sqrt{\frac{a_1}{2}}y} + (c_{19} + c_{20} y) e^{-\sqrt{\frac{a_1}{2}}y} + A \quad (19)$$

$$u = (b_{29} + b_{30} y) e^{q_1 y} + (b''_{29} + b''_{30} y) e^{-q_1 y} + A_6 y + A_7 \quad (20)$$

(ii) $a_1 < 0$

$$w = (c_{21} + c_{22} y) cos\sqrt{\frac{a_1}{2}}y + (c_{23} + c_{24} y) sin\sqrt{\frac{a_1}{2}}y + \frac{a_3}{a_2} \quad (21)$$

$$u = (b_{34} + b_{35} y) \sin q_1 y + (b_{36} + b_{37} y) \cos q_1 y + A_6 y + A_7 \quad (22)$$

$k < 0$, $k = -p^2$

$$w = e^{v_2 y}\left((c_{25} + c_{26} y) \cos v_1 y + (c_{27} + c_{28} y) \sin \; v_1 y\right) + \frac{a_3}{a_2} \quad (23)$$

$$u = (b_{72} + b_{73} y) \; e^{v_2 y} \cos v_1 y + (b_{36} + b_{37} y) \; e^{v_2 y} \sin v_1 y + A_6 y + A_7 \quad (24)$$

Case-II $\boldsymbol{a_1 = 0, a_2 \neq 0}$

$$w = e^{\sqrt{\frac{a_2}{2}}y}\left((c_{29} + c_{30}y)\cos\sqrt{\frac{a_2}{2}}y + (c_{31} + c_{32}y)sin\sqrt{\frac{a_2}{2}}y\right) + \frac{a_3}{a_2} \quad (25)$$

$$u = (b_{82}y + b_{83})\ e^{q_2y}\cos q_2y + (b_{84}y + b_{85})\ e^{q_2y}\sin q_2y + A_6y + A_7 \quad (26)$$

Case-III $\boldsymbol{a_1 \neq 0, a_2 = 0}$

$$w = c_{33}e^{\sqrt{a_1}y} + c_{34}e^{-\sqrt{a_1}y} + c_{35}cos\sqrt{a_1}y + c_{36}sin\sqrt{a_1}y - \frac{y^2\, a_3}{2\, a_1} \quad (27)$$

$$u = (b_{91} - c_{35})e^{ry} + b_{91}e^{-ry} + b_{92}\sin ry + b_{93}\cos ry + b_{94}y + A_7 \quad (28)$$

Case-IV $\boldsymbol{a_1 = 0, a_2 = 0}$

$$w = c_{37} + c_{38}y + c_{39}y^2 + c_{40}y^3 + \frac{y^4\, a_3}{24} \quad (29)$$

$$u = b_{95}y^3 + b_{96}y^2 + b_{97}y + b_{99} \quad (30)$$

The steady state solution of equation (8) is

$$\theta = 1 + (m - 1)y \quad (31)$$

The steady state solution of equation (9) is

$$C = 1 + (m_1 - 1)y \quad (32)$$

5. Numerical Results and Discussion of the problem

In this study, we have discussed effect of different physical parameter on velocity, angular velocity, temperature and concentration profiles for asymmetric and symmetric distribution of temperature and concentration of the vertical walls and presented their influences through Figs. 2-17. Even numbered figures are for asymmetric case ($\mathrm{m} = \mathrm{m}_1 = 0$) and rest is for symmetric case($\mathrm{m} = \mathrm{m}_1 = 1$).

Figure 2-3 displays the effect of magnetic parameter M and time t on the velocity profiles for asymmetric and symmetric distribution of temperature and concentration respectively. It is observed that the amplitude of the velocity as well as the boundary layer thickness decreases when M is increased. In both thermal cases, we observed that the velocity of fluid has attained steady state by increasing with time and steady state time 1.1, 0.9 and 0.7 for M = 0.1, M = 2 and M = 5 respectively under asymmetric case. Similarly, the steady state times are 1.2, 1.0 and 0.8 for M = 0.1, M = 2 and M = 5 respectively under symmetric case. The magnetic parameter M shows the accountable decreasing effect on the velocity profiles and on other hand, considerable decreasing effect observed on the steady state times for both cases (asymmetric and symmetric). Physically, increase the values of M, the drag force increases which leads to the deceleration of the flow.

Figure 4-5 indicates that the magnetic field parameter plays a important role in the micro-rotation regime for both cases. Further, in both cases, the magnitude of angular velocity and its steady state time decrease with increase the values of magnetic parameter M.

Figure 6-9 depict that effect of thermal Grashof number Gr and mass Grashof number Gm on velocity profile for asymmetric and symmetric distribution of temperature and concentration. The thermal

Grashof number Gr indicates the ratio of thermal buoyancy force to viscous force and mass Grashof number Gm means ratio of mass buoyancy force to viscous force. This implies that motion of fluid accelerated due to improvement in either temperature buoyancy force or mass buoyancy force. Physically, Increase in Gr indicates increase in the strength of the flow, small viscous effects in the momentum equation and thus, reasons the increase in velocity profiles.

Fig.10-13 depicts that effect of Gr and Gm on micro-rotation profile for both cases (asymmetric and symmetric). It is seen that micro-rotation profile increase with increase in Gr and Gm respectively.

Fig. 14-15 illustrate that temperature profile for different values of Prandtl number Pr for asymmetric and symmetric cases respectively. Physically, thermal boundary layer increases as Prandtl number decreases and hence temperature profile decreases with increase in Prandtl number.

Fig. 16-17 show concentration profile for different values of Schmidt number Sc. In both cases, we observed that Schmidt number Sc tends to reduced mass transfer process. Physically, the rise in the value of Schmidt number means reduction of molecular diffusion. Furthermore, it is interesting to note that the concentration profiles fall slowly and steadily for (Sc = 0. 22) and (Sc = 0.66) but falls very rapidly for (Sc = 1.5).

Table 1 depict that steady state solution of velocity and micro-rotation profile agrees with Ravi et al. [4] and Table 7-9 indicate that steady state solution of velocity, micro rotation and temperature profile agrees with Kataria et al. [8] respectively for both cases (asymmetric and symmetric).

6. Conclusion

The main conclusions of this study are as follows:

1. Magnitude of velocity, micro-rotation, temperature and concentration profiles is more in asymmetric case compared to symmetric case.
2. Magnitude of Velocity & Micro-rotation and its steady state time decrease with Magnetic Field M.
3. Thermal Grashof number Gr and mass Grashof number Gm tends to improve velocity and angular velocity also.
4. Prandtl number tends to reduced temperature profile whereas, Schmidt number tends to reduced concentration profile.
5. We conclude that steady state is achieved faster in case of asymmetric than symmetric case.
6. Effect of all parameters is similar in asymmetric $(\mathrm{m} = \mathrm{m}_1 = 0)$ distribution of temperature and concentration and symmetric $(\mathrm{m} = \mathrm{m}_1 = 1)$ distribution of temperature and concentration.

Appendix:

$A_4 = \frac{(1+R)(1+0.5R)}{RbM^2}$ $\quad A_5 = \frac{-(2+R)}{M^2}$ $\quad A_6 = \frac{(Gr(m-1)+Gm(m_1-1))}{M^2}$

$A_7 = \frac{Gm+Gr}{M^2}$ $\quad A' = A_6 + A_7$ $\quad a_1 = \frac{-Rb\,R}{(1+R)(1+0.5R)} +$

$\frac{2Rb}{(1+0.5R)}+\frac{M^2}{(1+R)}$

$a_2 = \frac{2M^2Rb}{(1+R)(1+0.5R)}$ $\quad a_3 = \frac{(Gr(1-m)+Gm(1-m_1))Rb}{(1+R)(1+0.5R)}$ $\quad$ k $= p^2 = {a_1}^2 - 4a_2$

$A = \frac{a_3}{a_2}$ $\quad p_1 = \frac{a1+p}{2}$ $\quad p_2 = \frac{a1-p}{2}$

$q_1 = \sqrt{\frac{a_1}{2}}$ $\quad q_2 = \sqrt{\frac{a_2}{2}}$ $\quad r = \sqrt{a_1}$

$v_1 = \sqrt{\frac{\sqrt{\frac{{a_1}^2}{4}+\frac{p^2}{4}}+\frac{a_1}{2}}{2}}$ $\quad v_2 = \sqrt{\frac{\sqrt{\frac{{a_1}^2}{4}+\frac{p^2}{4}}-\frac{a_1}{2}}{2}}$ $\quad b_1 = {p_1}^3 A_4$

$b_2 = {p_2}^3 A_4$ $\quad b_3 = p_1 A_5$ $\quad b_4 = A_5 p_2$

$b_5 = b_1 + b_3$ $\quad b_6 = b_2 + b_4$ $\quad b_7 = b_5(-A_1 - 1)$

$b_8 = b_5(-A_2 - 1)$ $\quad b_9 = b_5(-A_3 - A)$ $\quad b_{10} = c_3 b_7 + c_4 b_8 + b_9$

$b_{11} = A_1 c_3 b_5 + A_2 c_4 b_5 - A_3 b_5$ $\quad A_1 = \frac{e^{p_1}-e^{p_2}}{e^{-p_1}-e^{p_1}}$ $\quad A_2 = \frac{e^{p_1}-e^{-p_2}}{e^{-p_1}-e^{p_1}}$

$A_3 = \frac{A(e^{p_1}-1)}{e^{-p_1}-e^{p_1}}$ $\quad A_8 = -2b_5 A_1 - b_5 + b_6$ $\quad A_9 = -2b_5 A_2 - b_5 - b_6$

$A_{10} = -2b_5 A_3 - Ab_5 + A_7$ $\quad A_{11} = b_7 e^{p_1} - A_1 b_5 e^{-p_1} + b_6 e^{p_2}$ $\quad A_{12} = b_8 e^{p_1} - A_2 b_5 e^{-p_1} - b_6 e^{-p_2}$

$A_{13} = b_9 e^{p_1} - A_3 b_5 e^{-p_1} + A'$ $\quad c_4 = \frac{A_{13}A_8 - A_{10}A_{11}}{A_{11}A_9 - A_8 A_{12}}$ $\quad c_3 = \frac{A_{13}A_9 - A_{10}A_{12}}{A_{11}A_9 - A_8 A_{12}}$

$c_2 = A_1 c_3 + c_4 A_2 + A_3$ $\quad c_1 = -c_2 - c_3 - c_4 - A$ $\quad b_{12} = b_2 - b_4$

$b_{13} = b_5(-B_1 - 1)$ $\quad b_{14} = b_5(B_3 - A)$ $\quad b_{15} = c_7 b_{13} + c_8 B_2 b_5 + b_{14}$

$b_{16} = c_7 b_5 B_1 + c_8 b_5 B_2 + b_5 B_3$ $\quad B_1 = \frac{e^{p_1} - \cos p_2}{e^{-p_1}-e^{p_1}}$ $\quad B_2 = \frac{-\sin p_2}{e^{-p_1}-e^{p_1}}$

$B_3 = \frac{Ae^{p_1}-A}{e^{-p_1}-e^{p_1}}$ $\quad B_4 = -2b_5 B_1 - b_5$ $\quad B_5 = -b_2 + b_4$

$B_6 = -Ab_5 + A_7$ $\quad B_7 = b_{13}e^{p_1} - b_5 B_1 e^{-p_1} + b_{12}\sin p_2$ $\quad B_8 = b_5 e^{p_1} B_2 - b_5 B_2 e^{-p_1} - b_{12}\cos p_2$

$B_9 = b_{14}e^{p_1} - b_5 B_3 e^{-p_1} + A_6 + A_7$ $\quad c_8 = \frac{B_4 B_9 - B_6 B_7}{B_5 B_7 - B_8 B_4}$ $\quad c_7 = \frac{B_5 B_9 - B_6 B_8}{B_5 B_7 - B_8 B_4}$

$c_6 = B_1 c_7 + c_8 B_2 + B_3$ $\quad c_5 = -c_6 - c_7 - A$ $\quad b_{17} = b_1 - b_3$

$b_{18} = (-c_{11} - c_{12} - A)b_{17}$ $\quad b_{19} = b_{17}(-c_{11}B_{11} - c_{12}B_{12} - B_{13})$ $\quad B_{11} = \frac{1-e^{p_2}}{\sin p_1}$

$B_{12}=\frac{1-e^{-p_2}}{\sin p_1}$ $B_{13}=\frac{A(\cos p_1-1)}{\sin p_1}$ $B_{14}=-B_{11}b_{17}+b_6$

$B_{15}=-B_{12}b_{17}-b_6$ $B_{16}=-B_{13}b_{17}+A_7$ $B_{17}=(-\sin p_1-B_{11}\cos p_1)\,b_{17}+b_6e^{p_2}$

$B_{18}=(-\sin p_1-A_{12}\cos p_1)\,b_{17}-b_6e^{-p_2}$ $B_{19}=(-A\sin p_1-A_{13}\cos p_1)\,b_{17}+A'$ $c_{12}=\frac{B_{19}B_{15}-B_{16}B_{18}}{B_{14}B_{18}-B_{17}B_{15}}$

$c_{11}=\frac{B_{19}B_{15}-B_{16}B_{18}}{B_{15}B_{17}-B_{18}B_{14}}$ $c_{10}=B_{11}c_{11}+c_{12}B_{12}+B_{13}$ $c_9=-c_{11}-c_{12}-A$

$H_1=\frac{cos\,p_1-cos\,p_2}{\sin p_1}$ $H_2=\frac{-sin\,p_2}{\sin p_1}$ $H_3=\frac{Acos\,p_1-A}{\sin p_1}$

$b_{20}=(-c_{15}-A)b_{17}$ $b_{21}=b_{17}(-c_{15}H_1-H_3)$ $H_4=-H_1b_{17}$

$H_5=H_2b_{17}-b_{12}$ $H_6=H_3b_5+A_7$ $H_7=(-\sin p_1-H_1\cos p_1)b_{17}+b_{12}\sin p_2$

$H_8=H_2b_{17}\cos p_1-b_{12}\cos p_2$ $H_9=-Ab_{17}\sin p_1-b_{17}H_3\cos p_1+A'$ $c_{16}=\frac{H_9H_4-H_6H_7}{H_5H_7-H_8H_4}$

$c_{15}=\frac{H_9H_5-H_6H_8}{H_4H_8-H_7H_5}$ $c_{14}=H_1c_{15}-c_{16}H_2+H_3$ $c_{13}=-c_{15}-A$

$I_1=\frac{e^{q_1}-e^{-q_1}}{e^{q_1}}$ $I_2=\frac{e^{-q_1}}{e^{q_1}}$ $I_3=\frac{A(e^{q_1}-1)}{e^{q_1}}$

$b_{22}=q_1{}^3A_4$ $b_{23}=q_1{}^2A_4$ $b_{24}=q_1A_5$

$b_{25}=b_{22}+b_{24}$ $b_{26}=3b_{23}+A_5$ $b_{27}=b_{25}+b_{26}$

$b_{28}=b_{25}-b_{26}$ $b'_{29}=c_{19}I_1-c_{20}I_2+I_3$ $b_{29}=b_{25}(-c_{19}-A)+b_{26}b'_{29}$

$b_{30}=b_{25}b'_{29}$ $b''_{29}=-b_{25}c_{19}+b_{26}$ $b''_{30}=-c_{20}b_{25}$

$I_4=-2b_{25}+I_1b_{26}$ $I_5=-I_2b_{26}+b_{26}$ $I_6=-Ab_{22}+I_3b_{26}-Ab_{24}+A_7$

$I_7=-b_{25}(e^{q_1}+e^{-q_1})+I_1e^{q_1}b_{27}$ $I_8=-I_2e^{q_1}b_{27}-e^{-q_1}b_{28}$ $I_9=-Ae^{q_1}b_{25}+I_3e^{q_1}b_{27}+A_6+A_7$

$c_{20}=\frac{I_9I_4-I_6I_7}{I_5I_7-I_8I_4}$ $c_{19}=\frac{I_9I_5-I_6I_8}{I_4I_8-I_7I_5}$ $c_{18}=I_1c_{19}-c_{20}I_2+I_3$

$c_{17}=-c_{19}-A$ $J_1=-tanq_1$ $J_2=-tanq_1$

$J_3=\frac{\mathrm{A}(\cos q_1-1)}{\cos q_1}$ $b_{31}=3b_{23}-A_5$ $b_{32}=b_{22}-b_{24}$

$b_{33}=c_{23}J_1+c_{24}J_2+J_3$ $b_{34}=-Ab_{32}$ $b_{35}=b_{32}b_{33}$

$b_{36}=-b_{31}b_{33}-c_{23}b_{32}$ $b_{37}=-c_{24}b_{32}$ $J_4=-J_1b_{31}-b_{32}$

$J_5 = -J_2 b_{31}$ $\quad J_6 = -J_3 b_{31} + A_7$ $\quad J_6' = J_1 b_{31} - b_{32}$, $\quad J_7' = J_2 b_{32} - b_{31}$

$J_8' = J_2 b_{31} - b_{32}$ $\quad J_9' = -A b_{32} + J_3 b_{31}$ $\quad J_7 = \sin q_1 J_1 b_{32} - \cos q_1 J_6'$

$J_8 = \sin q_1 J_7' - \cos q_1 J_8'$ $\quad J_9 = \sin q_1 J_9' - J_3 \cos q_1 b_{31} + A'$ $\quad c_{24} = \frac{J_9 J_4 - J_6 J_7}{J_5 J_7 - J_8 J_4}$

$c_{23} = \frac{J_9 J_5 - J_6 J_8}{J_4 J_8 - J_7 J_5}$ $\quad c_{22} = J_1 c_{23} - c_{24} J_2 + J_3$ $\quad c_{21} = -A$

$M_1 = -\tan v_1$ $\quad M_2 = -\tan v_1$ $\quad M_3 = \frac{A(\cos v_1 e^{v_2} - 1)}{\cos v_1 e^{v_2}}$

$b_{38} = {v_2}^2 A_4$ $\quad b_{39} = {v_2}^2 v_1 A_4$ $\quad b_{40} = {v_1}^2 A_4$

$b_{41} = {v_1}^3 A_4$ $\quad b_{42} = v_1 v_2 A_4$ $\quad b_{43} = v_1 {v_2}^3 A_4$

$b_{44} = v_2 {v_1}^2 A_4$ $\quad b_{45} = v_1 A_5$ $\quad b_{46} = v_2 A_5$

$b_{46}' = {v_2}^3 A_4$ $\quad b_{47} = b_{38} - b_{40}$ $\quad b_{48} = 3b_{39} - b_{41}$

$b_{49} = 3b_{44} + b_{46}$ $\quad b_{50} = A(b_{46}' + b_{49})$ $\quad b_{51} = b_{41} - b_{45}$

$b_{52} = 3b_{47} - b_{50} + b_{46}' + A_5$ $\quad b_{53} = -b_{48} - 6b_{42} - b_{45}$ $\quad b_{54} = -b_{51} + 3b_{39}$

$b_{55} = b_{46}' - b_{50}$ $\quad b_{56} = M_1 b_{52} + b_{54}$ $\quad b_{57} = M_1 b_{53} + b_{55}$

$b_{58} = b_{54} + b_{42}$ $\quad b_{59} = b_{55} + 3b_{38} - 2b_{40} + A_5$ $\quad b_{60} = M_2 b_{52} + b_{58}$

$b_{61} = M_2 b_{53} + b_{59}$ $\quad b_{62} = b_{55} + (b_{46}' + A_5)$ $\quad b_{63} = A b_{50} - A b_{46}'$

$b_{64} = M_3 b_{62} + b_{63}$ $\quad b_{65} = M_3 b_{53} + A b_{54}$ $\quad b_{66} = 3b_{47} + A_5$

$b_{67} = 3b_{38} - 2b_{40} + A_5$ $\quad b_{68} = b_{48} + b_{45}$ $\quad b_{69} = M_1 c_{27} + M_2 c_{28} + M_3$

$b_{70} = b_{54} + b_{42} - A b_{55}$ $\quad b_{71} = b_{55} + b_{67} + A b_{68}$ $\quad b_{72} = b_{66} b_{69} + b_{70}$

$b_{73} = b_{55} b_{69} + b_{68}$ $\quad b_{74} = -b_{42} b_{69} + b_{71}$ $\quad b_{75} = -b_{68} b_{69} + b_{55}$

$M_4 = (3b_{47} + A_5) M_1 + b_{48} + b_{45}$ $\quad M_5 = (3b_{47} + A_5) M_2 + 6b_{42}$ $\quad M_6 = (3b_{47} + A_5) M_3 - b_{50} + A_7$

$M_7 = (\cos v_1 b_{56} + \sin v_1 b_{57}) e^{v_2}$ $\quad M_8 = e^{v_2} (\cos v_1 b_{60} + \sin v_1 b_{61})$ $\quad M_9 = e^{v_2} (\cos v_1 b_{64} + \sin v_1 b_{65})$

$c_{28} = \frac{M_9 M_4 - M_6 M_7}{M_5 M_7 - M_8 M_4}$ $\quad c_{27} = \frac{M_9 M_5 - M_6 M_8}{M_4 M_8 - M_7 M_5}$ $\quad c_{26} = M_1 c_{27} + c_{28} M_2 + M_3$

$c_{25} = -A$ $\quad L_1 = -\tan q_2$ $\quad L_2 = \frac{A(\cos q_2 e^{q_2} - 1)}{\cos q_2 e^{q_2}}$

$b_{77} = {q_2}^3 A_4$ $\quad b_{78} = q_2 A_5$ $\quad b_{79} = -2b_{77} + b_{78}$

$b_{80} = 2b_{77} + b_{78}$ $\quad b_{81} = L_1 b_{79} + L_1 A_5$ $\quad b_{81}' = b_{81} + b_{80}$

$b_{82}' = b_{82} + b_{79}$ $\quad b_{83}' = b_{81}' + 6b_{76}$ $\quad b_{84}' = b_{82}' + A_5$

$b'_{85} = b_{81} - Ab_{79}$ $b'_{86} = b_{82} + Ab_{80}$ $b'_{87} = c_{31} + c_{32}$

$b'_{87} = A(A_5 - b_{79})$ $b'_{88} = -6b'_{87}L_1 - 6A$ $b_{82} = -L_1 b_{80} - 6L_1 b_{76}$

$b_{83} = c_{31}b_{80} + L_1 A_5 b'_{87} + 6c_{32}b_{76} + b'_{87}$ $b_{84} = -b_{80}b_{81} + b_{79}$ $b_{85} = c_{31}b_{79} - b_{76}b'_{88} + c_{32}A_5 L_1$

$L_4 = 2b_{77} + A_5 L_1 + b_{79}$ $L_5 = 6b_{76} + A_5 L_1$ $L_6 = 2Ab_{77} - Ab_{79} + A_5 L_2 + A_7$

$L_7 = (\cos q_2 b'_{81} + \sin q_2 b'_{82})e^{q_2}$ $L_8 = (\cos q_2 b'_{83} + \sin q_2 b'_{84})e^{q_2}$ $L_9 = (\cos q_2 b'_{85} + \sin q_2 b'_{86})e^{q_2} + A'$

$c_{32} = \frac{L_9 L_4 - L_6 L_7}{L_5 L_7 - L_8 L_4}$ $c_{31} = \frac{L_9 L_5 - L_6 L_8}{L_4 L_8 - L_7 L_5}$ $c_{30} = L_1 c_{31} + c_{32} L_1 + L_2$

$c_{29} = -A$ $G_1 = \frac{\cos r - e^r}{e^r - e^{-r}}$ $G_2 = \frac{\sin r}{e^r - e^{-r}}$

$G_3 = \frac{(^{a_3}/_{2a_1})}{e^r - e^{-r}}$ $b_{86} = r^3 A_4$ $b_{87} = A_5 r$

$b_{88} = b_{86} + b_{87}$ $b_{89} = b_{86} - b_{87}$ $b_{90} = -G_1 c_{35} - G_2 c_{36} + G_3$

$b_{91} = b_{88}b_{90}$ $b_{92} = c_{35}b_{89}$ $b_{93} = c_{36}b_{89}$

$b_{94} = A_6 - \frac{a_3}{a_1}$ $G_4 = (-2G_1 - 1)b_{88}$ $G_5 = -2G_2 b_{88} - b_{89}$

$G_6 = 2G_3 b_{88} + A_7$ $G_7 = b_{88}(G_1(-e^r - e^{-r}) - e^r) + \sin r\, b_{89}$ $G_8 = b_{88}G_2(-e^r - e^{-r}) + \cos r\, b_{89}$

$G_9 = b_{88}G_3(-e^r - e^{-r}) - \frac{a_3}{a_1} + A'$ $c_{36} = \frac{G_9 G_4 - G_6 G_7}{G_5 G_7 - G_8 G_4}$ $c_{35} = \frac{G_9 G_5 - G_6 G_8}{G_4 G_8 - G_7 G_5}$

$c_{34} = G_1 c_{35} + c_{36} G_2 - G_3$ $c_{33} = -c_{34} - c_{35}$ $b_{95} = \frac{a_3}{6}$

$b_{96} = 3c_{40}$ $b_{97} = 2c_{39} + A_6 + A_4 a_3$ $b_{98} = c_{39} + c_{40} + \frac{a_3}{24}$

$b_{99} = 6c_{40}A_4 - b_{98}A_5 + A_7$ $D_1 = -A_5$, $D_2 = -6A_4$

$D_3 = \frac{-25a_3}{24} + A_7$ $D_4 = D_1 + 2$ $D_5 = D_2 + D_1 + 3$

$D_6 = A_4 c_{39} - \frac{A_5 a_3}{24} + b_{95} + A_6 + A_7$ $c_{40} = \frac{D_6 D_1 - D_3 D_4}{D_2 D_4 - D_5 D_1}$ $c_{39} = \frac{D_6 D_2 - D_3 D_5}{D_1 D_5 - D_4 D_2}$

$c_{38} = -c_{39} - c_{40} - \frac{a_3}{24}$ $c_{37} = 0$

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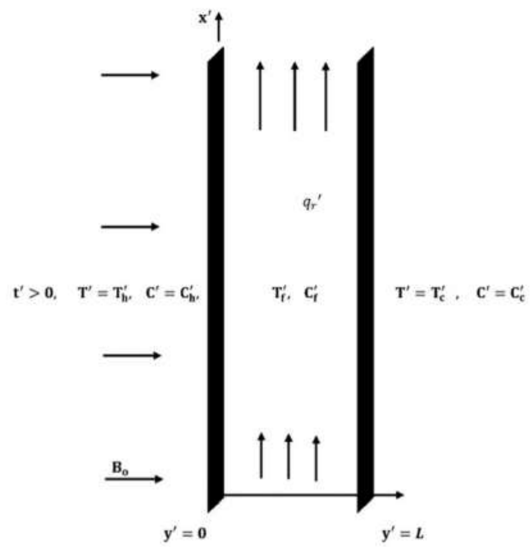

Figure 1: Physical sketch of the problem

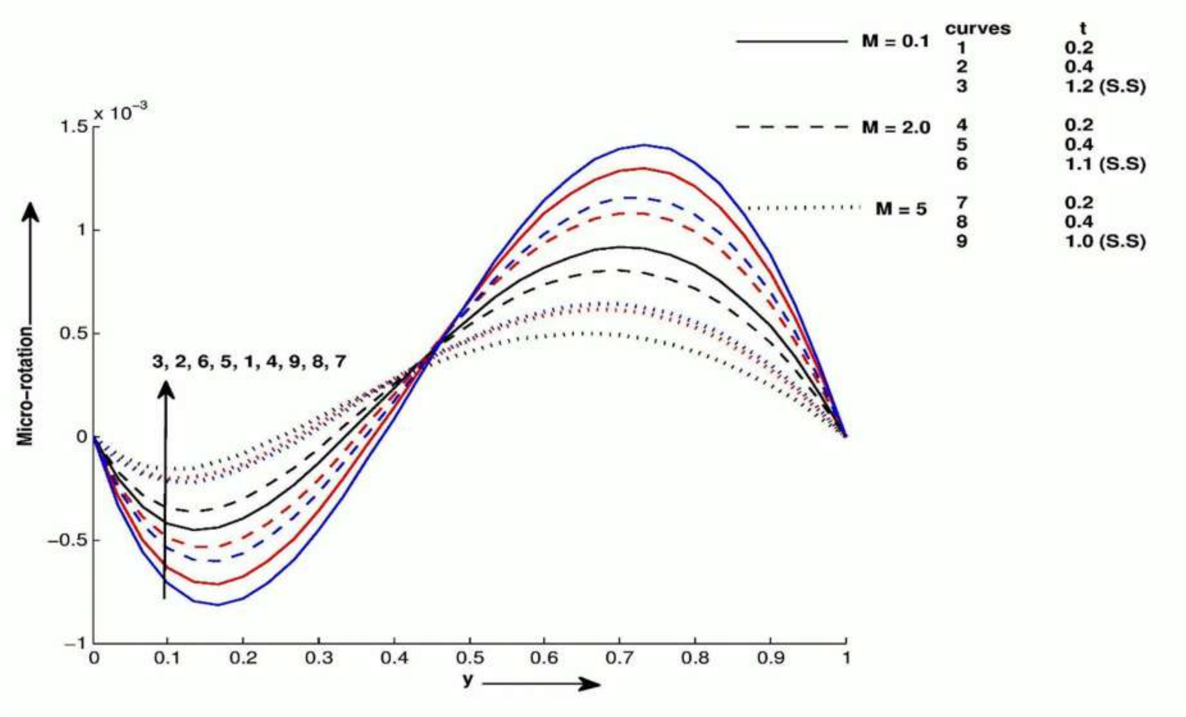

Figure 4: Micro-rotation profile w for different values of y & M at m = 0, $\mathbf{m_1 = 0}$, R = 0.5, b = 0.5, Sc =0.66, Gr = 1, Gm = 1 and Pr = 10

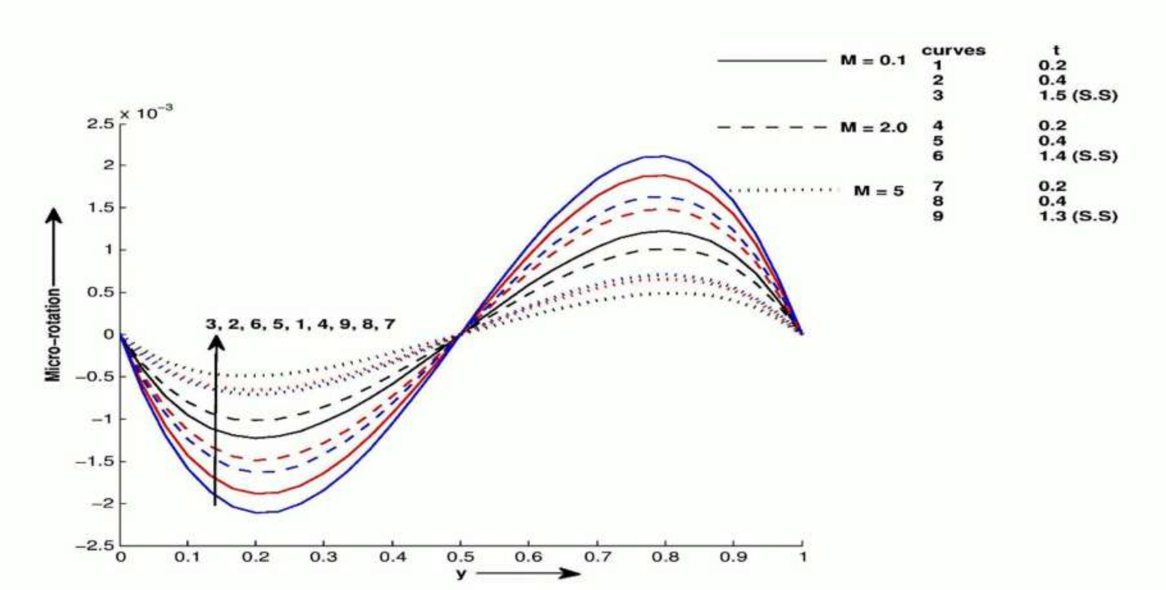

Figure 5: Micro-rotation profile w for different values of y & M at m = 1, $\mathbf{m_1 = 1}$, R = 0.5, b = 0.5, Sc =0.66, Gr = 1, Gm = 1 and Pr = 10

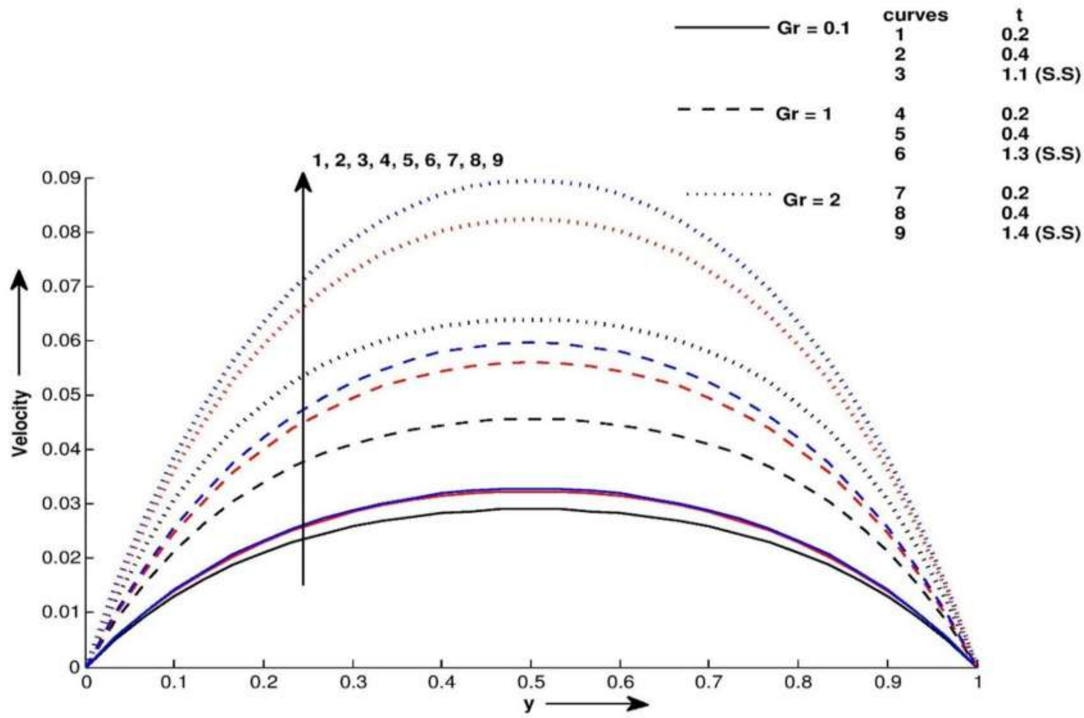

Figure 6: Velocity profile u for different values of y & Gr at m = 0, $\mathbf{m_1 = 0}$, R = 0.5, b = 0.5, M = 5, Sc = 0.66, Gm = 1 and Pr = 10

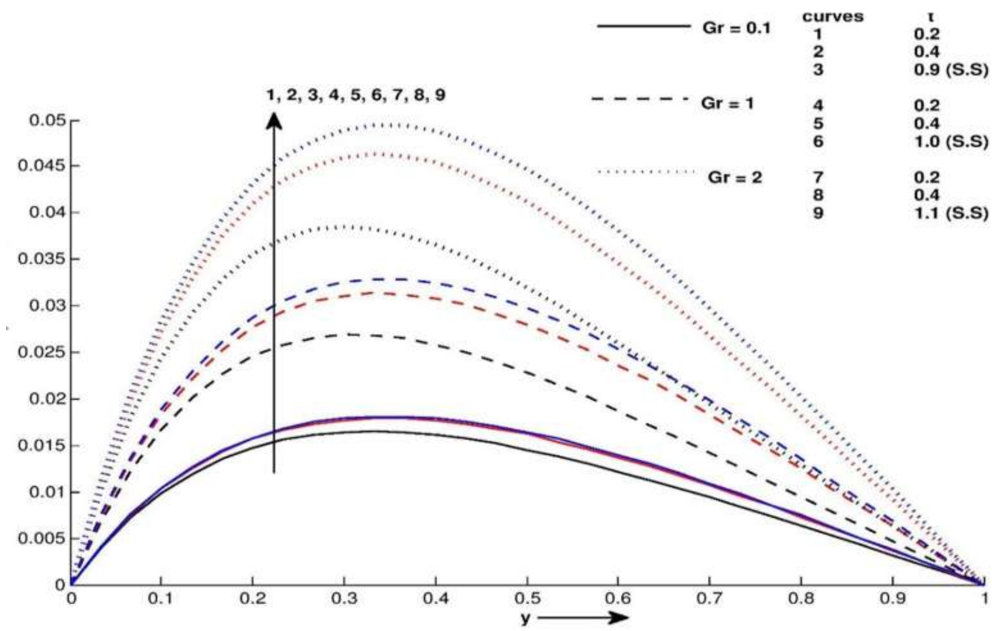

Figure 7: Velocity profile u for different values of y & Gr at m = 1, $\mathbf{m_1 = 1}$, R = 0.5, b = 0.5, M = 5, Sc = 0.66, Gm = 1 and Pr = 10

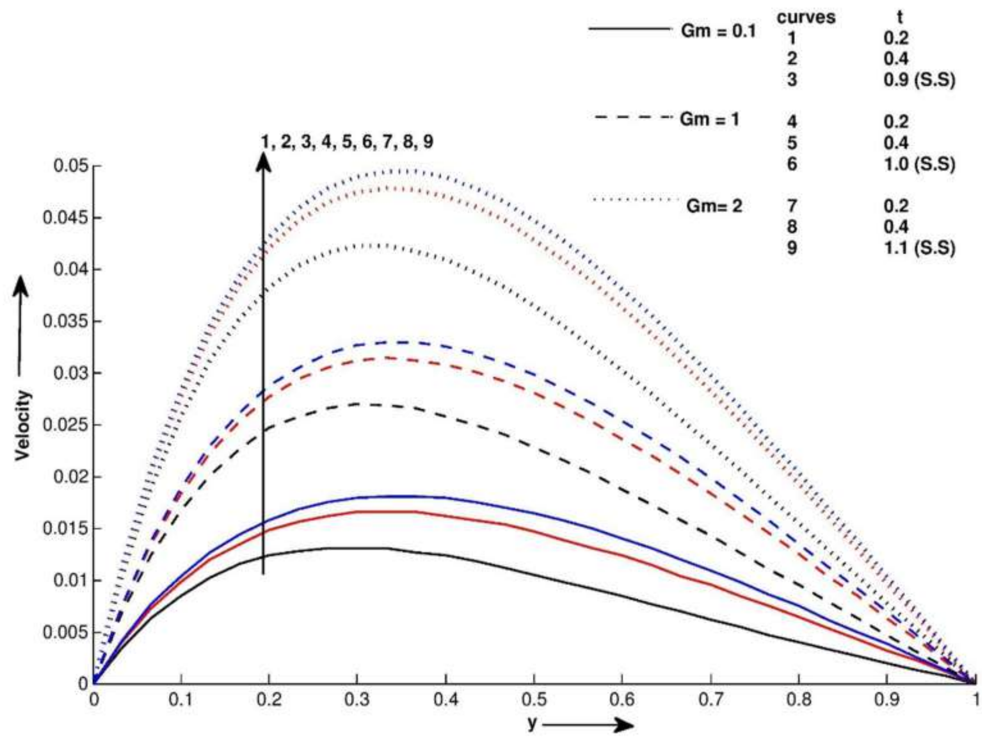

Figure 8: Velocity profile u for different values of y & Gm at m = 0, $\mathbf{m_1 = 0}$, R = 0.5, b = 0.5, M = 5, Sc = 0.66, Gr = 1 and Pr = 10

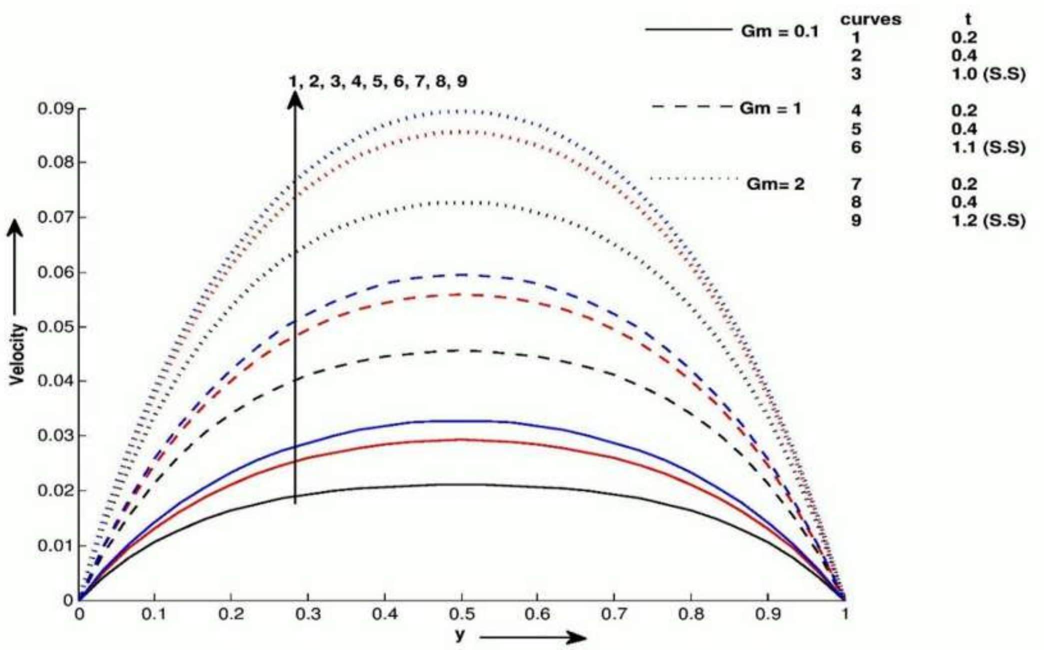

Figure 9: Velocity profile u for different values of y & Gm at m = 1, $m_1 = 1$, R = 0.5, b = 0.5, M = 5, Sc = 0.66, Gr = 1 and Pr = 10

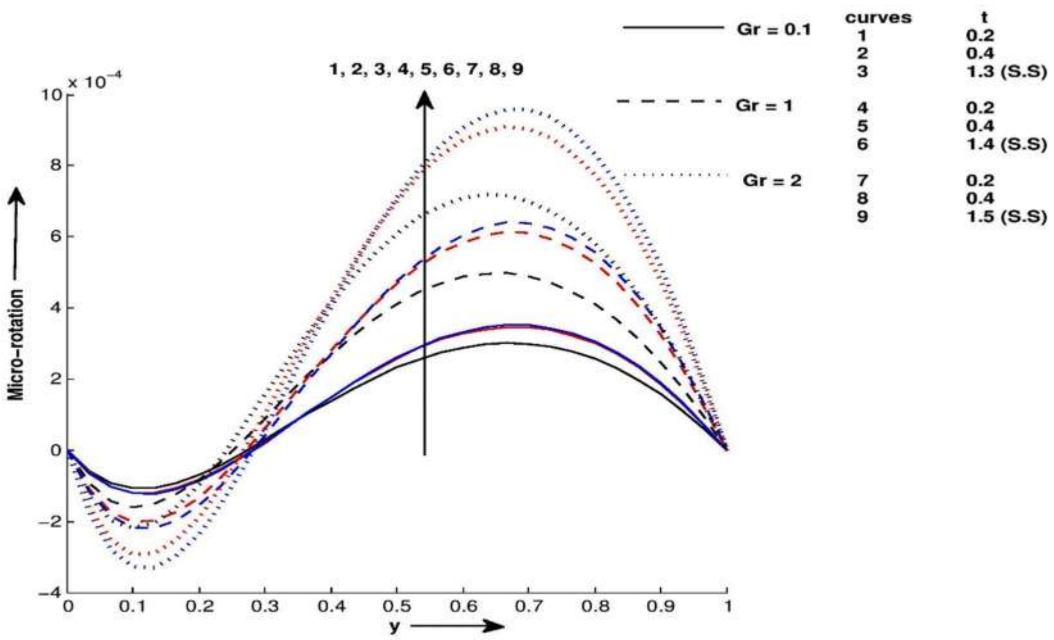

Figure 10: Micro-rotation profile w for different values of y & Gr at m = 0, $\mathbf{m_1 = 0}$, R = 0.5, b = 0.5, M = 5, Sc = 0.66, Gm = 1 and Pr = 10

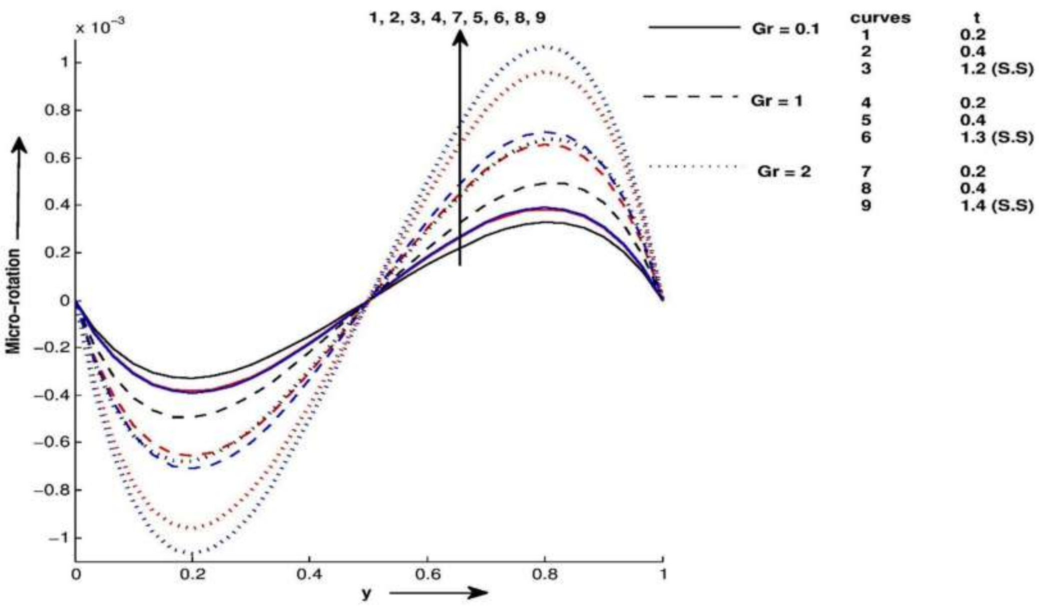

Figure 11: Micro-rotation profile w for different values of y & Gr at m = 1, $\mathbf{m_1 = 1}$, R = 0.5, b = 0.5, M = 5, Sc = 0.66, Gm = 1 and Pr = 10

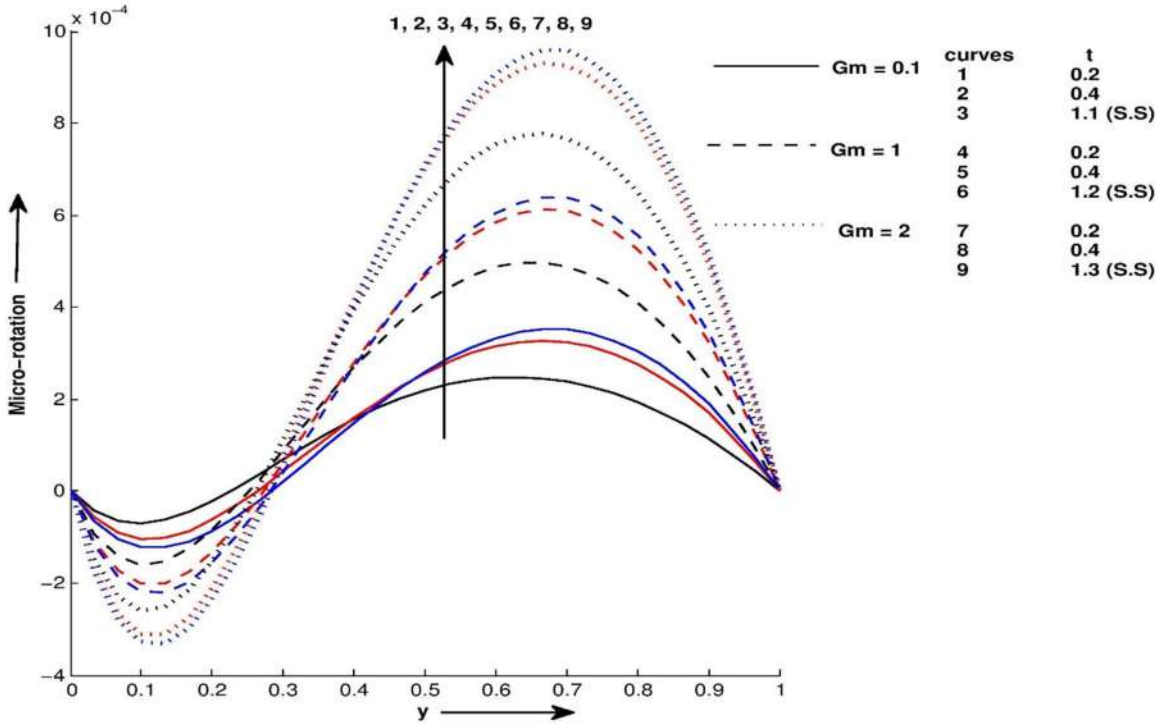

Figure 12: Micro-rotation profile w for different values of y & Gm at m = 0, $\mathbf{m_1 = 0}$, R = 0.5, b = 0.5, M = 5, Sc = 0.66, Gr = 1 and Pr = 10

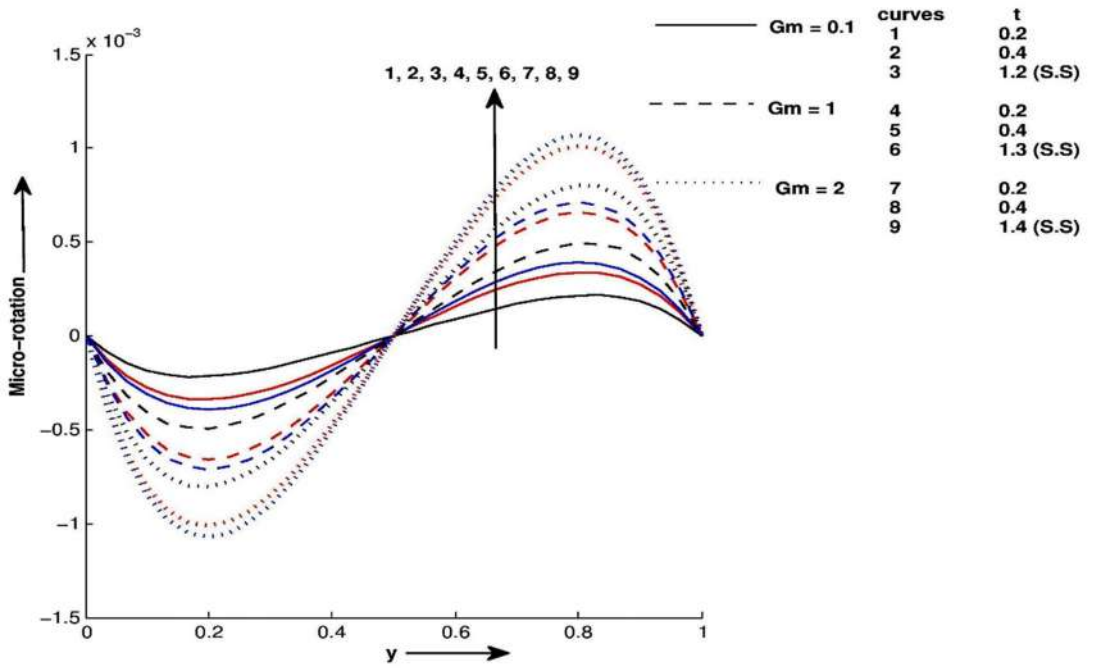

Figure 13: Micro-rotation profile w for different values of y & Gm at m = 1, $\mathbf{m_1 = 1}$, R = 0.5, b = 0.5, M = 5, Sc = 0.66, Gr = 1 and Pr = 10.

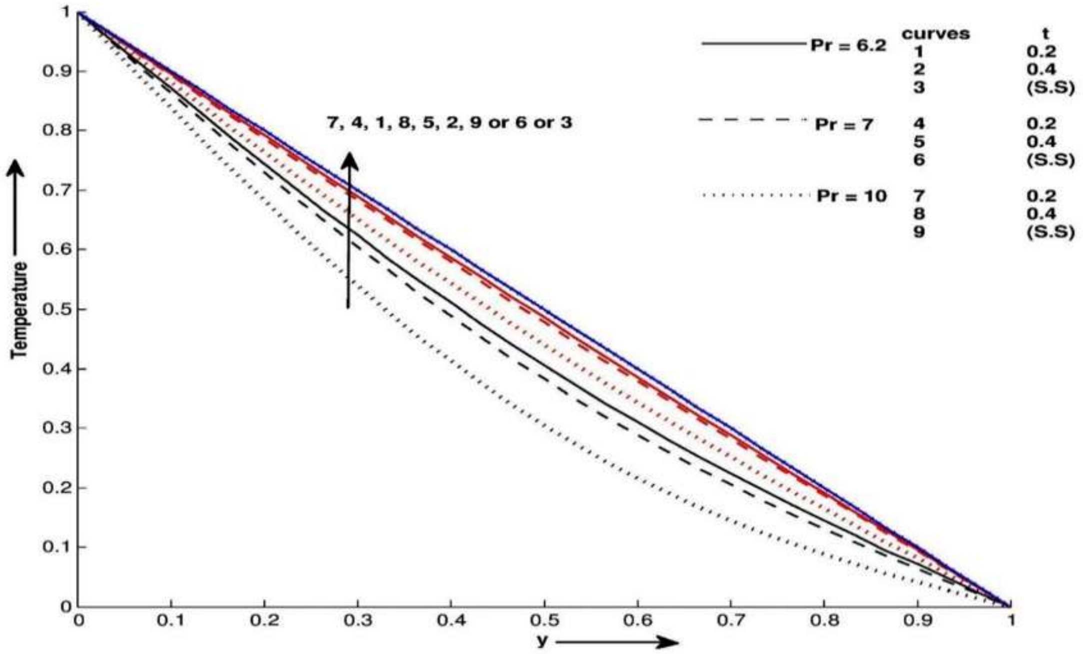

Figure 14: Temperature profile **θ** for different values of y & Pr at m = 0.

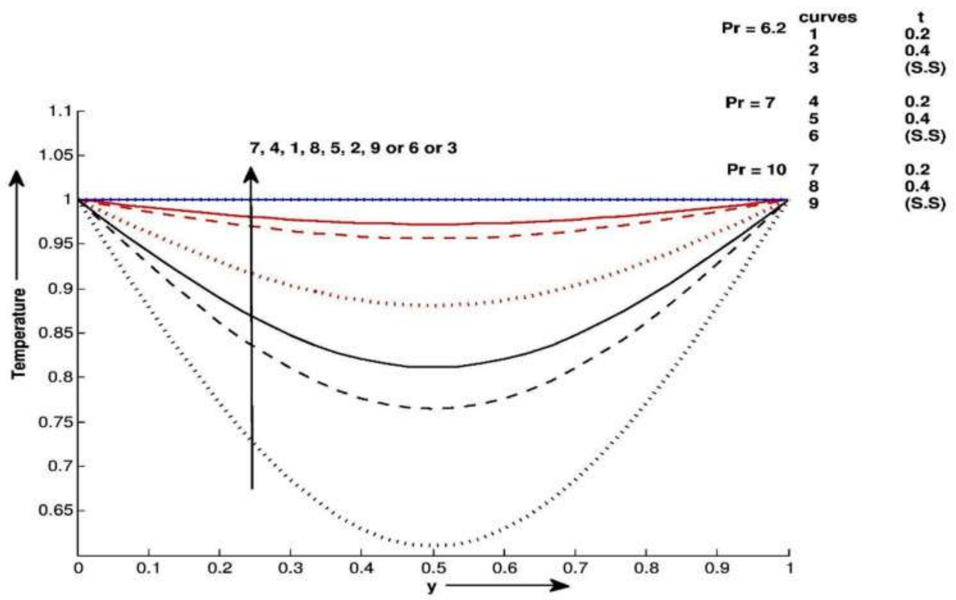

Figure 15: Temperature profile **θ** for different values of y & Pr at m = 1.

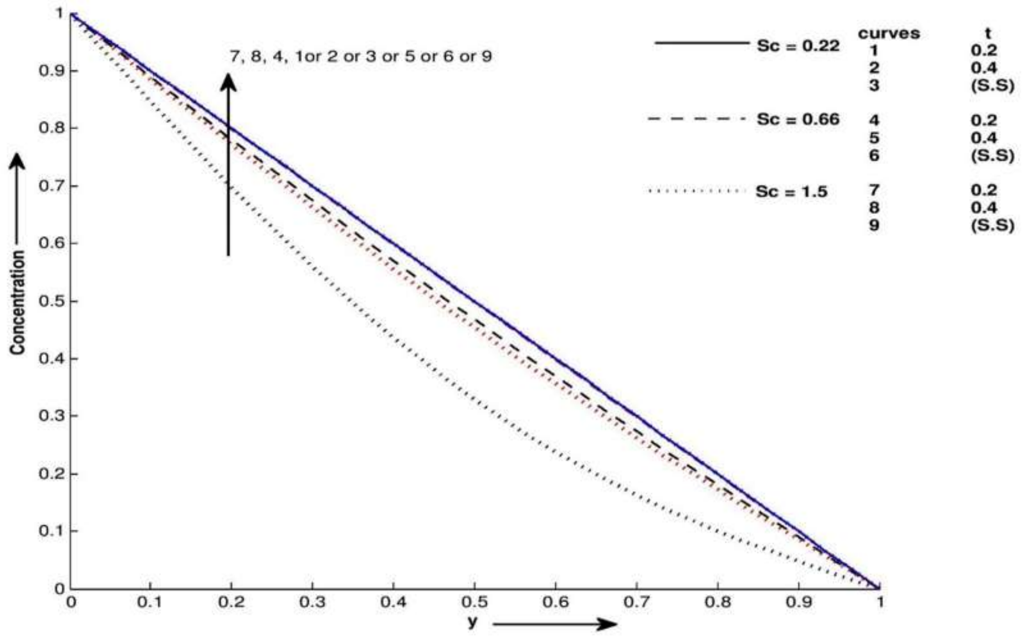

Figure 16: Concentration profile $\mathbf{C}$ for different values of y & Sc at $\mathbf{m_1 = 0}$

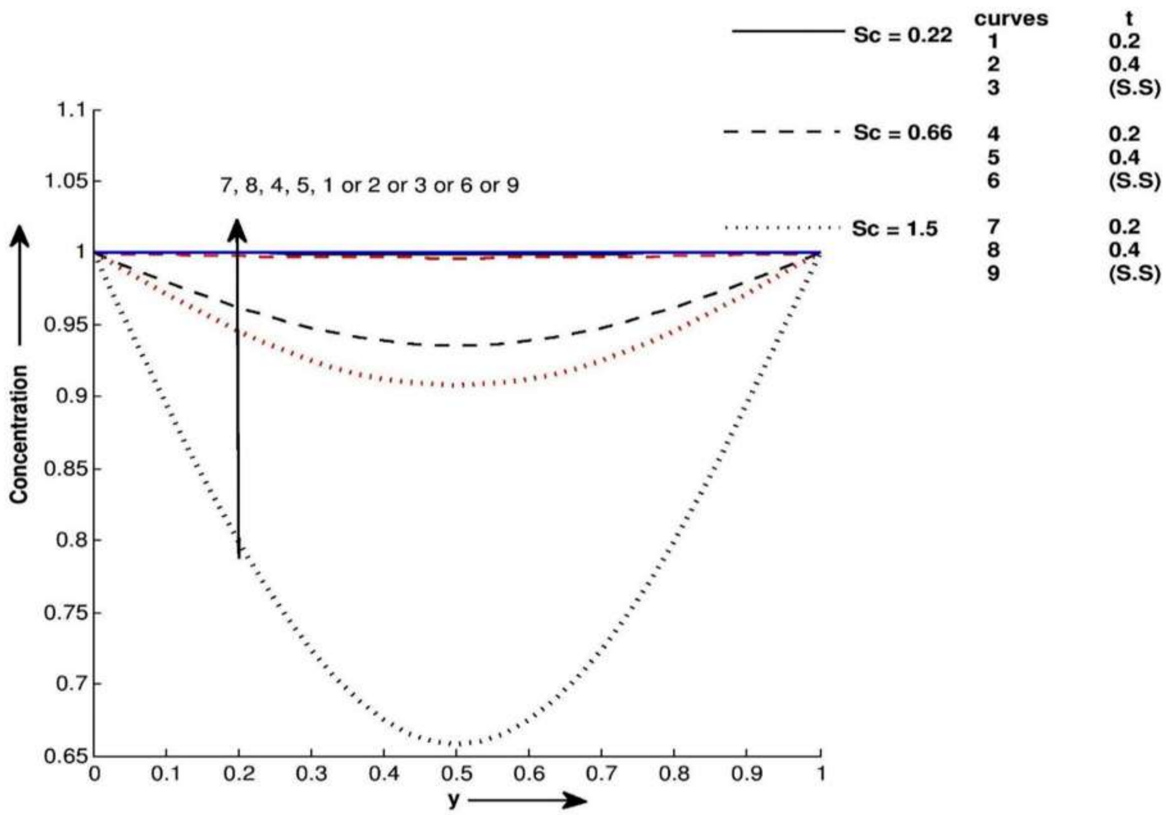

Figure 17: Concentration profile $\mathbf{C}$ for different values of y & Sc at $\mathbf{m_1 = 1}$

m	R	B	y	Analytic Result for (u) Ref.[4]	Analytic Result for (u)	Numerical Result for (u) Ref.[4]	Numerical Result for (u)	Analytic Result for (w)	Analytic Result for (w) Ref.[4]	Numerical Result for (w)	Numerical Result for (w). Ref [4]
0	0.5	1	0.0	0.0000	0.0000	0.0000	0.00000	0.00000	0.0000	0.0000	0.0000
			0.2	0.0320	0.0320	0.03206	0.03206	-0.00079	-0.0008	-0.00079	-0.00078
			0.4	0.0427	0.0428	0.04278	0.04278	0.0001	0.0001	0.00007	0.00010
			0.6	0.0374	0.0374	0.03742	0.03743	0.00111	0.0011	0.0011	0.00117
			0.8	0.0213	0.0213	0.02136	0.02136	0.00130	0.0013	0.0013	0.00135
			1.0	0.0000	0.0000	0.00000	0.00000	0.00000	0.0000	0.0000	0.0000
1	0.5	1	0.0	0.0000	0.0000	0.00000	0.00000	0.00000	0.0000	0.0000	0.0000
			0.2	0.0533	0.0534	0.05334	0.05343	-0.0021	-0.0021	-0.0021	-0.0021
			0.4	0.0800	0.0802	0.08021	0.08021	-0.0010	-0.0010	-0.0011	-0.0010
			0.6	0.0800	0.0802	0.08021	0.08021	0.0010	-0.0010	0.0010	0.0010
			0.8	0.0533	0.0534	0.05334	0.05343	0.0021	-0.0021	0.0021	0.0021
			1.0	0.0000	0.0000	0.00000	0.00000	0.0000	0.0000	0.0000	0.0000

Table 1: Comparison of Velocity and Micro-rotation profiles with Ref. [4], at $\mathbf{m_1} = \mathbf{0}$, M = 0, Pr = 1, Sc = 0, Gr = 1 and Gm = 0

M	Pr	m	R	b	y	Analytic Solution for (u), Ref [8]	Analytic Solution for (u),	Numerical Solution for (u), Ref.[8]	Numerical Solution for (u)
0.1	1.0	0	0.5	0.1	0.2	0.0320	0.0320	0.0319	0.0320
					0.6	0.0373	0.0373	0.0373	0.0373
5	1.0	0	0.5	0.1	0.2	0.0143	0.0143	0.0144	0.0144
					0.6	0.0127	0.0127	0.0127	0.0127
0.1	1.0	1	0.5	0.1	0.2	0.0533	0.0533	0.0533	0.0533
					0.6	0.0800	0.0800	0.0800	0.0800
5	1.0	1	0.5	0.1	0.2	0.0211	0.0211	0.0211	0.0211
					0.6	0.0289	0.0289	0.0290	0.0290

Table 2: Comparison of velocity profile with Ref. [8], at $\mathbf{m_1} = \mathbf{0}$ and Sc = 0.

M	Pr	m	R	b	y	Analytic Solution for (w) Ref.[8]	Analytic Solution for (w)	Numerical Solution for (w), Ref.[8]	Numerical Solution for (w)
0.1	1.0	0	0.5	0.1	0.2	-0.0000783	-0.000078	-0.0000784	-0.0000784
					0.6	0.0001167	0.0001167	0.0001164	0.0001164
5	1.0	0	0.5	0.1	0.2	-0.0000149	-0.000149	-0.00001517	-0.0000151
					0.6	0.0000619	0.0000619	0.00006189	0.0000618
0.1	1.0	1	0.5	0.1	0.2	-0.0002128	-0.000212	-0.0002128	-0.0002128
					0.6	0.0001064	0.0001064	0.0001064	0.0001064
5	1.0	1	0.5	0.1	0.2	-0.0000715	-0.000071	-0.00007175	-0.0000717
					0.6	0.0000337	0.0000337	0.00003377	0.0000337

Table 3: Comparison of micro-rotation profile with Ref. [8], at $\mathbf{m_1 = 0}$ **and** Sc = 0.

M	Pr	m	R	b	y	Analytic Solution for (θ) Ref. [8]	Analytic Solution for (θ)	Numerical Solution for (θ), Ref. [8]	Numerical Solution for (θ)
0.1	1.0	0	0.5	0.1	0.2	0.8000	0.8000	0.8000	0.8000
					0.6	0.4000	0.4000	0.4000	0.4000
5	1.0	0	0.5	0.1	0.2	0.8000	0.8000	0.8000	0.8000
					0.6	0.4000	0.4000	0.4000	0.4000
0.1	1.0	1	0.5	0.1	0.2	1	1	1	1
					0.6	1	1	1	1
5	1.0	1	0.5	0.1	0.2	1	1	1	1
					0.6	1	1	1	1

Table 4: Comparison of temperature profile with Ref. [8].