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Numerical Analysis of Water Solidification in a Rectangular Encloser using for Ice Storage Systems

P. Bhargavi*, Radha Gupta** and K. Rama Narasimha***

*Research Scholar, Jain University, Bengaluru, Karnataka, India. bhargavinataraj@gmail.com

** Department of Mathematics, Dayananda Sagar College of Engineering,

Bengaluru, Karnataka, India. radha.guarav.gupta@gmail.com

***Department of Mechanical Engineering, K.S. Institute of Technology

Bengaluru, Karnataka, India. k.ramanarasimha@gmail.com

Abstract

Phase transition in processes is an area of great technological importance in many fields. The characteristic feature of these processes is presence of time evolving unknown boundaries separating the phases. Latent heat release at the interface during the process is used to store the thermal energy. As one of the most important energy technologies to balance the utilization of electricity power, ice storage is developing rapidly in recent years.Present study concentrates on continuous change in the depths of the frozen and dehydrated regions, describing solidification phenomenon in a rectangular region which studies the dynamics of the moving interphases, using an independent heat balance equation.Quasi-stable approximations for temperature distribution lead to a semi-analytical solution for the model. MATLAB software is used to solve the system of equations using the finite difference approximations. Numerical simulations are carried out varying surface temperature and initial freezing temperature on the dehydrated front, sublimation front, and also on the sublimation and frozen region temperatures.Results show that the effect of surface temperature is more than the effect of initial freezing temperature on the frozen zone.

Keywords: Water solidification, surface temperature, moving boundaries, ice storage.

AMS Subject Classification (2010): 80A20

1. Introduction

Freezing is a phase change process in which liquid turns to solid. Freezing or melting problems are referred as moving boundary problems as there evolve boundaries separating the regions. These boundaries move with constant rate.The prototype of such problems is the Stefan problem, named after the early work of J. Stefan, who studied the melting of the polar ice cap around 1890. Moving boundary problems are mathematically particularly interesting and difficult because the changing time history precludes the use of a similarity variable which is commonly emphasized in traditional Stefan problems to facilitate their solution. Establishing and maintaining of a constant front velocity is of crucial importance because constant flux rates are desirable in many modern technological procedures.Such a process covers a wide range of applications in which phase changes from solid, liquid or vapor states. Tracking the moving boundary at any time is an important part of the solution. Owing to the unknown location of the phase change interface and the nonlinear form of the thermal energy balance equation at the interface, analytical solutions are difficult to obtain except for a limited number of special cases.

Thermal energy storage (TES) involves adding heat energy to a storage medium, and then removing it from that medium for use at some other time. This may involve storing thermal energy at high temperatures (heat storage) or at low temperatures (cool storage). An ice storage system, however, uses the latent capacity of water, associated with changing phase from a solid (ice) to a liquid (water), to store thermal energy. The most-common storage media used for cool thermal storage are ice and water. Water is an ideal choice for thermal storage systems because of its availability, high latent heat, non-flammability, and nontoxicity. A chilled-water storage systemuses sensible-heat capacity of large volume of water to store thermal energy. Solidification and melting problems account for control based modelling. Solidification phenomena play a dominant role in the processes as well as product optimization.

In this work, one-dimensional model for controlling the solidification rate of pure water by controlling the process parameters isdeveloped. A time dependent upper boundary condition is incorporated in the model at the surface of the upper boundary. Very limited research is reported with time varying boundary condition for the phase change processes.

Solidification modeling with constant flux is reported in numerous papers and books. A wide research has been carried out in order to describe phenomena at microscale levels. Due to difficulties in obtaining analytical solutions, various numerical techniques are often employed [1]. Gupta and Banik [2] have investigated approximate analytical methods that yield solutions of Stefan problems in simple closed forms. Time varying surface temperature condition is applied to solve the integral equation to obtain the position of the interface in melting or solidification of a semi-infinite medium[3].

Numerical techniques are specially known to have difficulties with time-dependent boundary conditions, and very fine mesh size and small time steps are often needed for accurate solutions. Because these are often computer intensive—only a few results for the Stefan problem with time dependent boundary conditions are available in the literature. A comparative study of various numerical methods for moving boundary problems discussed in the paper by Furzeland [4].Solutions of such Stefan problems include linear, exponentialand periodical variation of the surface temperature or the flux with time [4–6].Solutions reported in the literature using the finite difference methods for solving the moving boundary problems include the one-dimensional Stefan problem describing the evaporation processes [6,7].

Changes in the soil depths due to the seasonal effects are studied in [8]. The variation of depths of the frozen and thawed soils is reduced to a moving boundary problem. The frozen and thawed depth moving boundaries are governed by the Stefan condition at the interface andto find the total ice content,is described by an independent mass balance equation. A periodic sinusoidal upper boundary condition for temperature is applied tostudy it's on the frost/thaw depths and soil temperature, by varying soil thickness, ground surface temperature, annual amplitude of ground surface temperature and thermal conductivity.A moving boundary problem of solidification of Lava lake to find the positions of the two moving interfaces is discussed in the paper [9]. A convective heat- transfer from the surface of lava-lake into the atmosphere and a conductive heat transfer in to the country rock from base is taken into account. Semi-analytical solution is obtained using Fourier series method where in Fourier spectral approach is used to obtain the solution in the spatial domain. Modified finite difference scheme is used in the time domain. Time dependent boundary conditions are described at the contact with the country rock for the problem. Appropriate Fourier sine series expansion for the temperature distribution which satisfies the boundary conditions reduces the heat transfer equation in to a system of first order ODEs.To study shoreline movement, a shoreline model for the sedimentary ocean basin is developed by [10]. This is a Stefan problem for melting, in which latent heat is treated as a linear function of space and only the active liquid phase is taken into count. The main focus of this paper is to understand the surface processes interact with changes in sea level. This problem differs with the classic Stefan problem of melting by adopting a fixed flux and variable latent heat at the boundary x=0.

Freezing with surface ice sublimation is studied to predict the effect of system parameters on the moving fronts and the temperature distributions to predict the feasible conditions for the coupled heat and mass transfer problem [11].Study is extended in paper [12], by introducing the mass transfer coefficient that describes the amount of vapor distribution from the surface of the dehydrated region to the atmosphere. The effect of mass transfer coefficient and the effect of the frozen mass volume are studied, to study their effect on the frozen and dehydrated region depths, in which sublimation is taken into account in the dehydrated region. One dimensional solidification in a rectangular encloserthat are used for ice storage units is discussed in the paper[13], that describes the effect of Stephen number on the solidification thickness and the velocity of the moving front. A quasi- stable temperature distribution is taken in to account by considering linear and quadratic approximations for the temperature distribution in the solid region. Extending the work reported by [13], by taking cubic and exponential approximations for the temperature distribution in the solid region [14]. Comparative study is made to report the best suitable approximation for the faster ice growth in rectangular ice storage systems.

To control the solidification process by controlling the process parameters in their permissible limits is the main objective of the paper. The heat balance equations are formulated for each subdomain. This approach leads to one partial differential equation (PDE) for each subdomain (phase), with one boundary condition and one interface condition. The interface, or moving boundary, yields an ODE for the position of the interface. Sublimated region along with the frozen region is taken into account to predict the vapor diffusion in to the air at the air-water interface. This approach is very important to decide the final quality of the product. A rectangular column of length 'L' is considered with water as the medium for the one dimensional phase change problem. The domain of interest is divided in to three regions, sublimated, frozen and unfrozen regions, defined by the phase change temperature. Sublimated and frozen regions are taken into account to describe the phenomenon. Each phase equations and suitable boundary conditions with interface condition explain the process and model equations are solved using finite difference method. A convective upper boundary condition at x=0 is considered as a function of time. Surface temperature and the initial freezing temperature are varied to study their effect on the solidification rate. This will help in the construction of ice storage systems to meet the energy requirement whenever is needed.

2. Mathematical Modelling and Governing Equations

Differential equations at the dehydrated region:

$$\rho_1 C_1 \frac{\partial T_1}{\partial t} = k_1 \frac{\partial^2 T_1}{\partial x^2}, \; 0 < x < s_1(t), \quad t > 0 \tag{1}$$

Differential equations at the frozen region:

$$\rho_2 C_2 \frac{\partial T_2}{\partial t} = k_2 \frac{\partial^2 T_2}{\partial x^2}, \; s_1(t) < x < s_2(t), \qquad t > 0 \tag{2}$$

Free boundary conditions at the moving sublimation front $x = s_1(t)$**:**

$$T_1(s_1(t), t) = T_2(s_1(t), t) = T_{sub}(t) \tag{3}$$

$$k_2 \frac{\partial T_2(s_1(t))}{\partial x} - k_1 \frac{\partial T_1(s_1(t))}{\partial x} = L_1 m_1 \dot{s_1}(t) \tag{4}$$

Free boundary conditions at the moving freezing front $x = s_2(t)$**:**

$$T_2(s_2(t), t) = T_{if}, \; t > 0 \tag{5}$$

$$k_2 \frac{\partial T_2(s_2(t), t)}{\partial x} = m_2 L_2 \dot{s_2}(t), \; t > 0 \tag{6}$$

$T_{sub}(t)$, T_{if} are sublimation temperature and initial freezing temperatures.

Convective boundary conditions at the fixed interface $x = 0$**:**

$$k_1 \frac{\partial T_1(0,t)}{\partial x} = h(T_1(0,t) - T_s), \qquad t > 0 \tag{7}$$

Initial conditions at $t = 0$**:**

$$s_1(0) = s_2(0) = 0 \tag{8}$$

$$T = T_{if} \text{ for } x \geq 0 \tag{9}$$

T is temperature, t is time, x is the length of the column. $s_1(t)$, $s_2(t)$, $T_1(x,t)$, $T_2(x,t)$ are the dehydrated and frozen depths and temperatures, respectively. L_1, L_2, m_1, m_2, c_1, c_2, ρ_1, ρ_2 k_1, k_2 are latent heat , mass per unit volume, volumetric heat capacity, density and thermal conductivity of dehydrated and frozen regions, respectively. h **,** is the heat transfer coefficient of water.

We assume temperature at the upper boundary to be:

$$T_1(0,t) = f(t) = T_{if} + \pi e^{-t} \tag{10}$$

By assuming quasi-steady approximation for T_1, T_2

$$T_1(x,t) = A(t) + B(t)x\,, \quad 0 < x < s_1(t),\ t > 0 \tag{11}$$

$$T_2(x,t) = C(t) + D(t)x\,,\ s_1(t) < x < s_2(t)\ t > 0 \tag{12}$$

Using initial and boundary conditions we evaluate the constants as follows

$$A(t) = T_{sub}(t) - s_1(t)(\frac{h}{k_1}(f(t) - T_s) \tag{13}$$

$$B(t) = \frac{h}{k_1}(f(t) - T_s) \tag{14}$$

$$c(t) = \frac{T_{sub}(t)s_2(t) - T_{if}s_1(t)}{s_2(t) - s_1(t)} \tag{15}$$

$$D(t) = \frac{T_{if} - T_{sub}(t)}{s_2(t) - s_1(t)} \tag{16}$$

Performing, mathematical calculations we get the following first order simultaneous differential equations for the two moving fronts, $s_1(t)$, $s_2(t)$.

$$\dot{s}_2(t) = \frac{k_2 T_{if}}{m_2 L_2}[\frac{(1 - \frac{T_{sub}(t)}{T_{if}})}{(s_2(t) - (\frac{k_1}{h}\frac{(T_{sub}(t) - f(t))}{(f(t) - T_s)}))}] \tag{17}$$

$$\dot{s}_1(t) = \frac{m_2 L_2}{L_1 m_1}\dot{s}_2(t) - \frac{h}{L_1 m_1}(f(t) - T_s) \tag{18}$$

Choosing the following non-dimensional parameters as follows

$$\delta_1 = \frac{m_2 L_2}{k_2 T_{if}}\ ,\ \delta_2 = \frac{L_1 m_1}{h T_{if}},\ \delta_3 = \frac{L_1 m_1}{k_2 T_{if}},\ \delta_4 = \frac{T_s}{T_{if}},\ \delta_5 = \frac{T_{sub}(t)}{T_{if}}$$

Equations (17) and (18) are reduced to the following non-dimensional form

$$\dot{s}_2(t) = [\frac{(1 - \delta_5)}{\delta_1(s_2(t) - (\frac{k_1}{h}\frac{(T_{sub}(t) - f(t))}{(f(t) - T_s)}))}] \tag{19}$$

$$\dot{s}_1(t) = \frac{\delta_1}{\delta_3}\dot{s}_2(t) - \frac{1}{\delta_2}\left(\frac{f(t)}{T_{if}} - \delta_4\right) \tag{20}$$

$$s_2(0) = s_1(0) = 0 \tag{21}$$

The quasi stable distribution for the temperature in the frozen and dehydrated regions reduces the PDE to system of ODE. Equation (19), (20) are simultaneous ordinary differential equations which are solved using finite difference approximations for the position of the moving fronts $s_1(t)$ and $s_2(t)$. Using the initial conditions for the position of the moving fronts, model equations are solved and are simulated using MATLAB software to study the effect of the process parameters surrounding and initial freezing temperatures.

Expressions for $T_1(x,t)$ & $T_2(x,t)$ are as follows

$$T_1(x,t) = T_{sub}(t) + \frac{h}{k_1}(x - s_1(t))((f(t) - T_s)\ 0 < x < s_1(t),\ \ t > 0 \tag{22}$$

$$T_2(x,t) = T_{if}[1 - \delta_1 \dot{s}_2(t)(s_2(t) - x)]\ s_d(t) < x < s_f(t)\ t > 0 \tag{23}$$

3. Results and Discussions

Solidification phenomenon of water analyzed with different surrounding temperature for 10,000 seconds at 1 atm pressure, where the initial temperature of water was considered to be 273K. The values of the physical properties and other parameters in frozen and unfrozen zones are same as in the referred journal [11]. We vary the surrounding temperature between $T_s = 233$ to $T_s = 263$, initial freezing temperature in the range of $T_{if} = 273$ to $T_{if} = 303$ and analyze the growth of dehydrated and frozen regions.

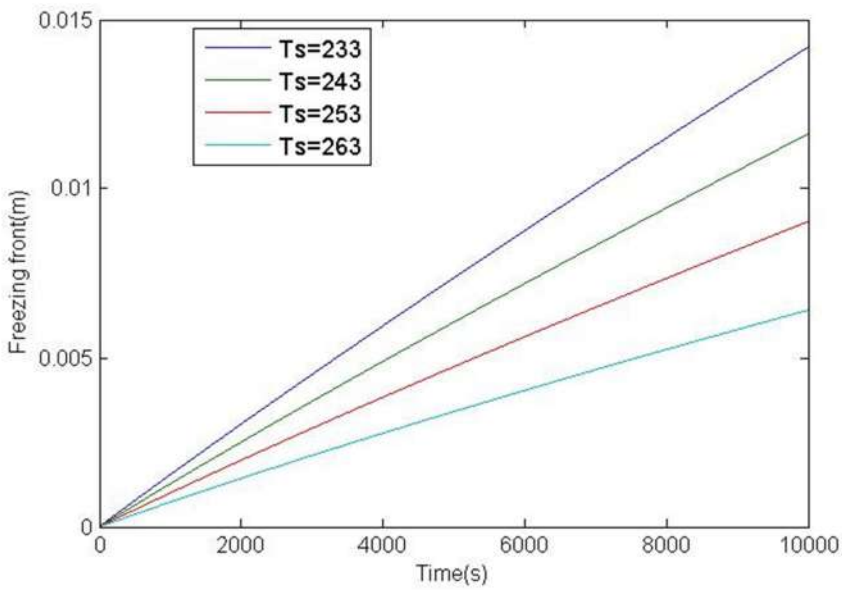

Figure1: Influence of the surrounding temperature in the evolution of the freezing front, $h = 75W/(m^2K)$, $T_{if} = 273.16$

Figure 1 shows that the decreasing of surrounding temperature affects ice thickness growth in frozen region. Lower is the surrounding temperature, higher is the growth rate. Ice thickness growth in the frozen region is linearly increasing for the increasing time period. The ice growth is more in the initial time period, because of the formation of ice crystals, rising to the sublimated interface and sticking there. In pure water, during freezing, the onset of freezing refers, the time during which crystal growth is occurring. Fast freezing rates like reducing the surrounding temperatures promote the formation of many small ice crystals during this period. Until all the freezable water has crystallizes, the partially frozen mixture will not cool further. Hence the growth of freezing front moves slowly during this time period. For certain period growth rate becomes almost nil and then increases at later times. Lower the value of the surrounding temperatures, faster the crystal formation, which in turn the movement of freezing front will be faster. The effect of initial freezing temperature on the freezing front is shown in the figure 2.

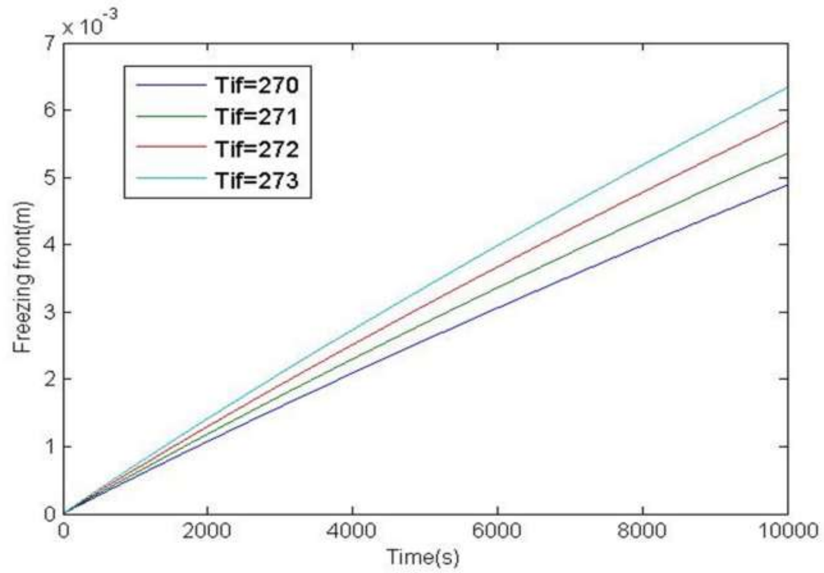

Figure2: Influence of the initial freezing temperature in the evolution of the freezing front, $h = 75W/(m^2K)$**,** $T_s = 263$**.**

The order of magnitude of growth rate is very less for the variation of initial freezing temperature, when compared to the variation of the surrounding temperature. We notice that when the initial freezing temperature is at 273K, the growth rate is more compared to the values of T_{if} at lower temperatures.

During freezing of the aqueous solution, a freeze-concentration process occurs as water freezes in the form of pure ice crystals. Hence the freezing temperature of the remaining solution drops. Some water remains at temperatures below the initial freezing point. Also, a large increase in the viscosity of the unfrozen phase occurs, thus decreasing the diffusion properties of the system and hindering crystallization. The study indicates that reducing initial freezing temperature will not improve the growth rate of the freezing front. So the effect of surrounding temperature is more dominant for the formation of ice quickly in the frozen region than the effect of the initial freezing temperature.

To predict the effect of surrounding temperature for ice thickness in the dehydrated region, the numerical solution has been run again with decreasing surrounding temperatures. For this part of modeling, results show that dehydrated region growth is lesser compare to frozen region growth for decreasing surrounding temperatures. The reason for this is that once the liquid region reaches T_{if}, the convection effect disappears, and only conduction heat transfer takes place in the solid region. So that, conduction heat transfer plays a dominant role in the overall freezing process. Sublimation of water in the dehydrated region into the atmosphere in the form of vapor is dominated by convection heat transfer mechanism, this process is slow in the dehydrated region than in the frozen region. This hinders the growth of dehydrated region for the same surrounding temperature variations.

Figure 3 shows that the decreasing surrounding temperature from 263K-233K decreases the growth rate and there is no significant variation in the growth of sublimation front for different T_s. Near the beginning of the freezing process, thickness of ice is very thin and has little impact on heat transfer. As freezing progresses, however, the ice becomes thicker and significantly impedes heat transfer. In order to maintain the same freeze rate with this degrading heat transfer, the temperature of the fluid must decrease near the end of the freezing process.Temperature variation during the freezing process has a close relationship with the rate of freezing, and it would affect the size of the ice crystal nuclei and the quality of the freezing process. In order to investigate the correlation between the initial freezing temperature and the water freezing process, the initial freezing temperature variation with respect to the process time was determined. Same trend can be observed from figure 4 for the variation of T_{if} from 270K -273K on the growth of sublimation region. Both T_s and T_{if} variation has increased the sublimation front growth of same order of magnitude. So we can conclude that no significant effect is observed on the growth rate of dehydrated region for different initial freezing temperature and surrounding temperatures.

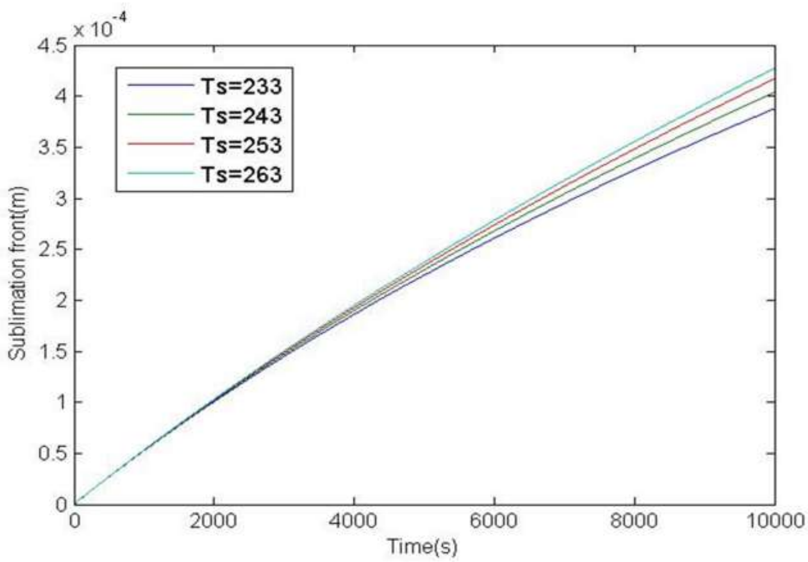

Figure3: Influence of the surrounding temperature in the evolution of the sublimation front, $h = 75W/(m^2K)$, $T_{if} = 273.16$.

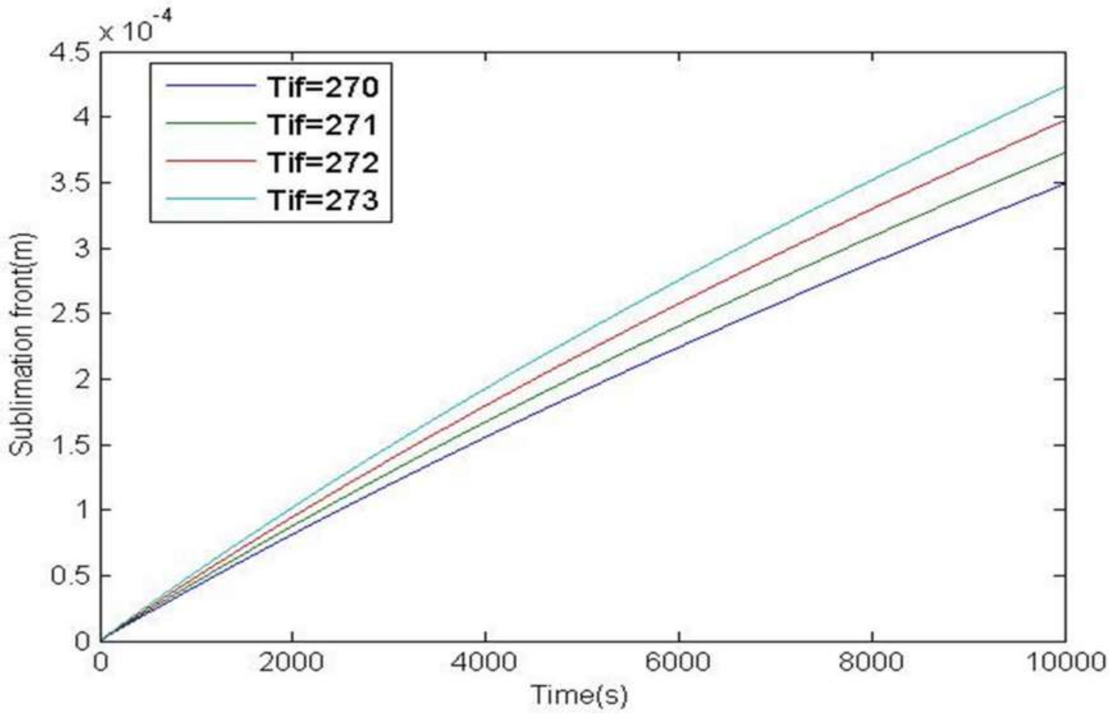

Figure4: Influence of the initial freezing temperature in the evolution of the sublimation front, $h = 75W/(m^2K)$, $T_s = 263$.

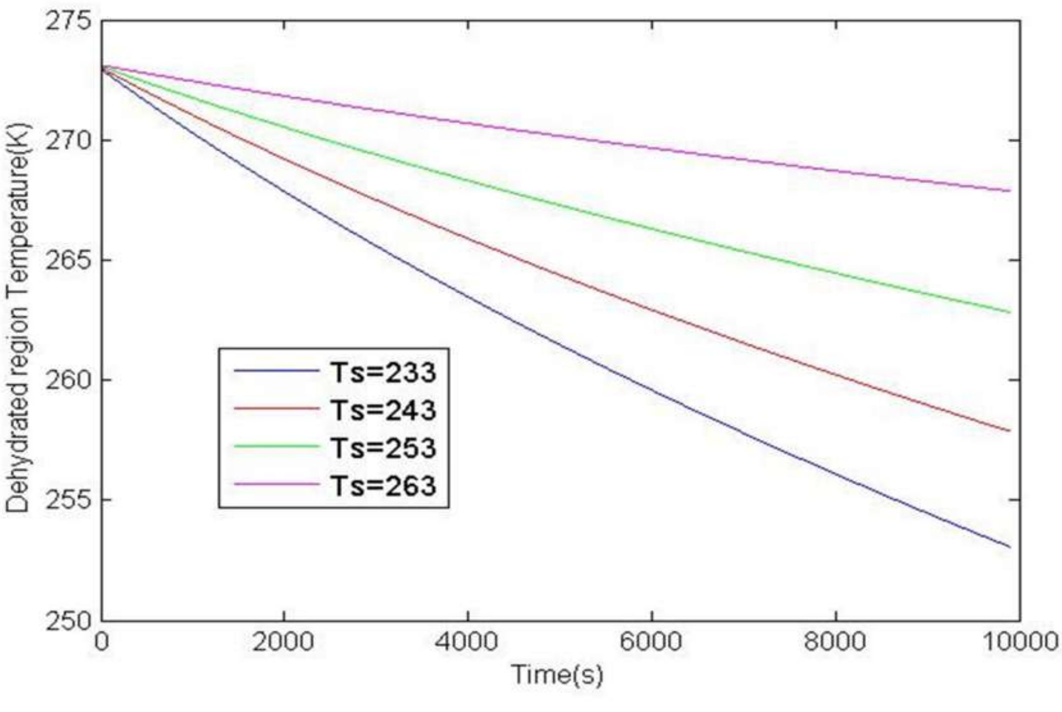

Figure5: Influence of the surrounding temperature in the evolution of the dehydrated region Temperature, $h = 75W/(m^2K)$, $T_{if} = 273.16$.

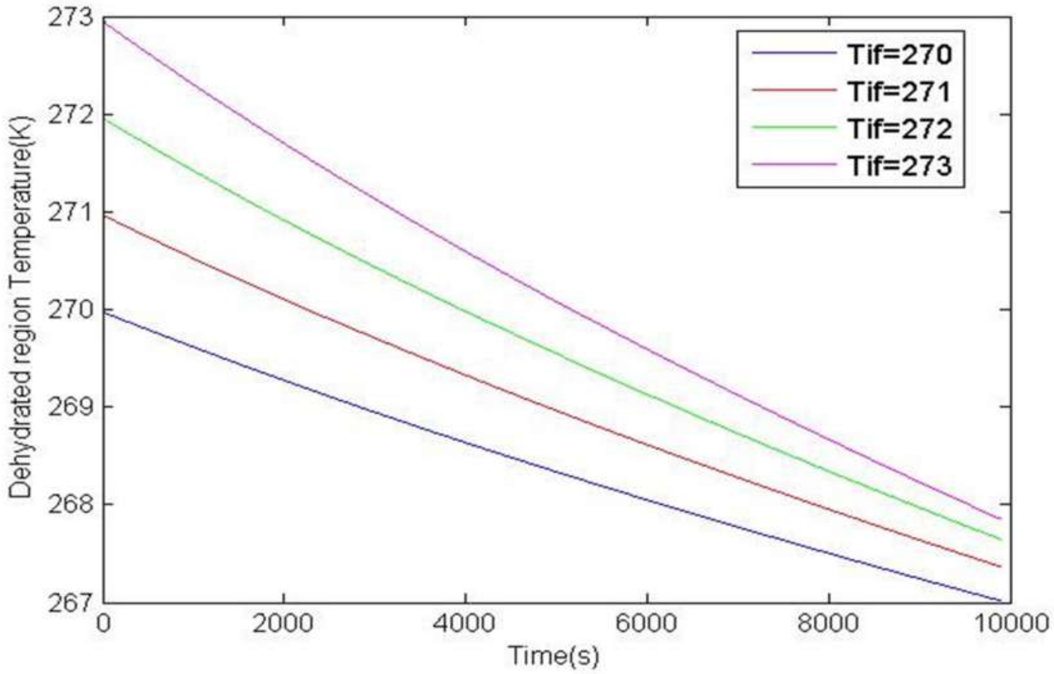

Figure6: Influence of the initial freezing temperature in the evolution of the dehydrated region Temperature, $h = 75W/(m^2K)$, $T_s = 263$.

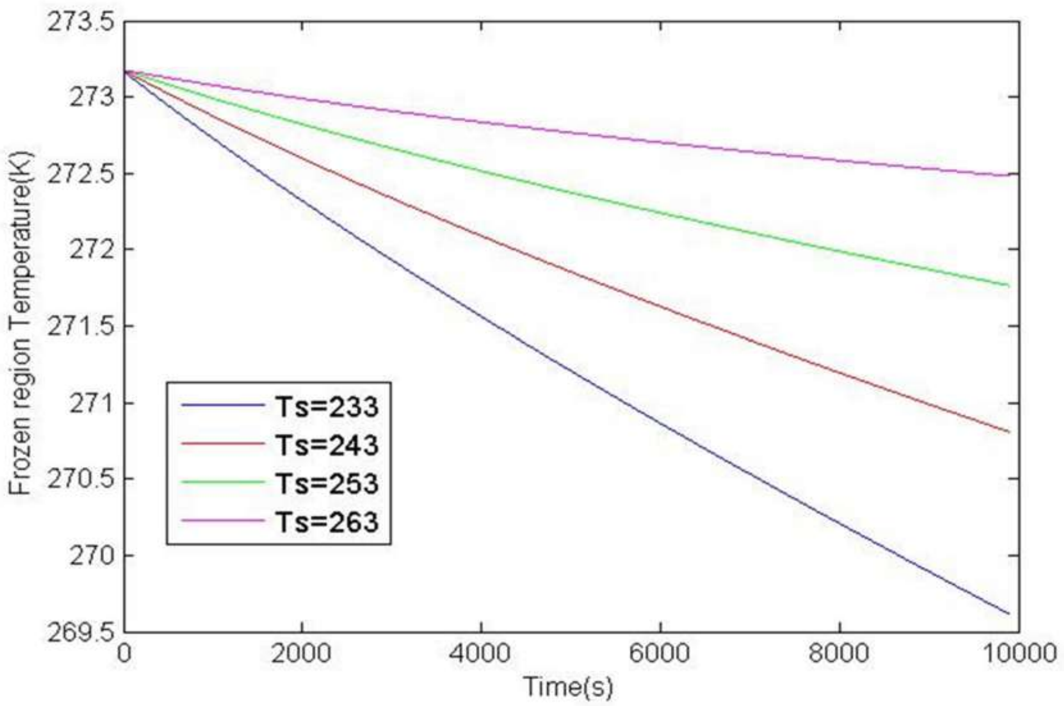

Figure7: Influence of the surrounding temperature in the evolution of the frozen region temperature, $h = 75W/(m^2K)$, $T_{if} = 273.16$.

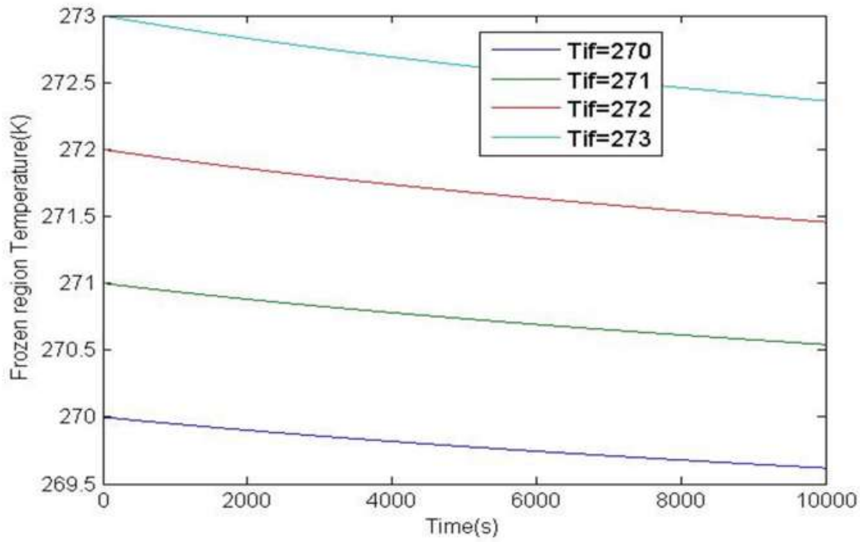

Figure8: Influence of the initial freezing temperature in the evolution of the frozen region temperature, $h = 75W/(m^2K)$, $T_{if} = 273.16$.

The effect of surrounding temperature (T_s) and the effect of initial freezing temperature (T_{if}) on the dehydrated region temperature can be seen in figure 5 and in figure 6. Even though the growth rate of dehydrated region is almost same for different T_s and T_{if}, temperature decrease in the dehydrated region is more significant for variation in T_s. Dehydrated region temperature is dominated by the variation of T_s, than T_{if}. Temperature has decreased from a maximum of 273K to minimum of 253K for various T_s. But the same has reduced from a maximum of 273K to minimum of 267K for various T_{if}. Temperature in the dehydrated region is decreasing with decreasing T_s and T_{if}.The same trend is observed even for frozen region temperature (figure 8). The effect of T_s is more on the frozen region temperature, than on the dehydrated region temperature.

4. Conclusion

The objective of this study was to conduct research on heat transfer in a specific ice storage system which utilizes rectangular ice containers and to develop models which can be used to simulate and evaluate the performance of the thermal energy storage system. The effects of process parameters like surrounding temperature and initial freezing temperature on the moving boundaries of sublimated and frozen regions were analyzed. The solution, is semi analytic, is sufficiently accurate for engineering design and prediction of ice accumulation. Results show that decreasing surroundingtemperature increases the growth of frozen region compared to dehydrated region. This study demonstrates the effectiveness of the surrounding and initial freezing temperaturesfor diagnosing and optimizing the process of water freezing to save energy.This can predict ice thickness for designing an ice storage tank./

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